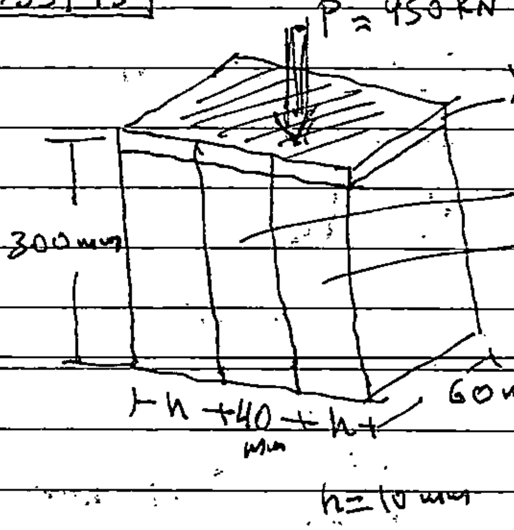
Π.Κ/Π

Επίλυση Προβλημάτων Αντοχής Υλικών.

Βιβλίο Beer et al Μηχανική των Υλικών.

ΚΕΦ 2°

2.33/95



Ανάμητη πλάκα

πυρήνας από Ορείχαλκο $E_{op} = 105$ GPa

Πλάκες αλουμινίου $E_{Al} = 70$ GPa

Υπολ. την ορθή τάση

(a) στον ορείχαλκο

(b) στο Αλουμίνιο

Απ. $P = P_{op} + A_{Al} = 450$ (1) Κατανομή του φορτίου

Λόγω της αναμητης πλάκας: $\Delta l_{op} = \Delta l_{Al} \Rightarrow \frac{P_{op} L_{op}}{A_{op} E_{op}} = \frac{P_{Al} \cdot L_{Al}}{A_{Al} \cdot E_{Al}} \Rightarrow$

$P_{op} = P_{Al} \cdot \frac{A_{op} E_{op}}{A_{Al} E_{Al}}$ (2)

Διατομές: $A_{op} = 40 \times 60 = 2400$ mm^2

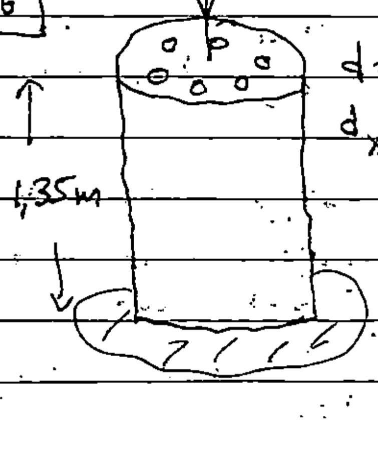
$A_{Al} = 10 \times 60 = 600$ mm^2 $\xrightarrow{\times 2}$ 1200 mm^2

$\therefore \frac{A_{op}}{A_{Al}} = 2$ & $\frac{E_{op}}{E_{Al}} = \frac{105}{70} = 1{,}5 \Rightarrow \frac{(AE)_{op}}{(AE)_{Al}} = 3$

$\therefore P_{op} = 3 P_{Al}$ (3)

(1) & (3) $\to 4 P_{Al} = 450 \to$ $P_{Al} = 112{,}5$ kN, $P_{op} = 337{,}5$ kN

2.35/96



$d = 0{,}45$ m $\quad E_{χαλ} = 200$ GPa

$d_{χαλ} = 28$ mm $\quad E_{σκ} = 29$ GPa

Υπολ. τις ορθές τάσεις σε χάλυβα κ' σκυρόδεμα

Απ. $P = 1560 = 6 P_{χαλ} + P_{σκυρ}$

$\Delta l_{χαλ} = \Delta l_{σκυρ} \to \frac{P_{χαλ} \cdot l_{χαλ}}{(AE)_{χαλ}} = \frac{P_{σκυρ} \cdot l_{σκυρ}}{(AE)_{σκυρ}}$

$$P_{\chi\alpha\lambda} = P_{\sigma\upsilon\nu\rho} \frac{(AE)_{\chi\alpha\lambda}}{(AE)_{\sigma\upsilon\nu\rho}}$$

	d mm	A mm²	E, GPa	AE	$\frac{(AE)_\chi}{(AE)_{\sigma\upsilon\nu}}$
Χαλ.	28	615,44	200	123088	2,67
Συνρ.	45	1589,62	29	46099	

$$P_{\chi\alpha\lambda} = P_{\sigma\upsilon\nu\rho} \frac{(AE)_\chi}{(AE)_{\sigma\upsilon}} = 2{,}67\, P_{\sigma\upsilon\nu\rho}$$

$$\therefore\ 1560 = P_{\sigma\upsilon\nu\rho} + (2{,}67\, P_{\sigma\upsilon\nu\rho})6 \longrightarrow P_{\sigma\upsilon\nu\rho} = 91{,}65\ kN$$

$$P_{\chi\alpha\lambda} \simeq 244{,}72\ kN$$

2.36/96 $\sigma_{\chi\alpha\lambda} = 138$ MPa

$\sigma_{\sigma\upsilon\nu\rho} = 16$ MPa. Υπολ. $P = ?$

Απ.

$$\sigma_{\chi\alpha\lambda} = 138\ \frac{N}{mm^2} = \frac{P_{\chi\alpha\lambda}}{A_{\chi\alpha\lambda}} \Rightarrow P_{\chi\alpha\lambda} = 84930\ N = 84{,}9\ kN$$

$$A_{\chi\alpha\lambda} \to \pi\frac{28^2}{4} = 615{,}44\ mm^2$$

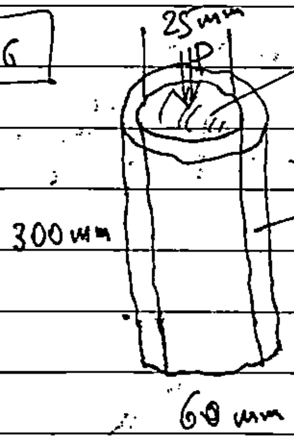
$$\sigma_{\sigma\upsilon\nu\rho} = 16 = \frac{P_{\sigma\upsilon\nu\rho}}{A_{\sigma\upsilon\nu\rho}} \Rightarrow P_{\sigma\upsilon\nu\rho} = 25434\ kN$$

$A_{\sigma\upsilon\nu\rho} \to 1589{,}62$

$$\therefore\ P = 6P_{\chi\alpha\lambda} + P_{\sigma\upsilon\nu\rho} = 535014\ N \Rightarrow \boxed{P \cong 535\ kN}$$

↑ # μπετόβεργες

2.38/96



ορείχαλκος $E = 105$ GPa

Αλουμ. $E = 70$ GPa

$\Delta l = 40$ mm

μείωση μήκους δι εφαρμογή φορτίου σε άκαμπτη πλάκα στην κορυφή.

Υπ. (a) την P

(b) $\sigma_{o\rho}$ & $\sigma_{A\lambda}$

Απ. $P = P_{o\rho} + P_{A\lambda}$ (1)

$$\Delta l_{o\rho} = \Delta l_{A\lambda} \to \frac{P_{o\rho}\, l_{o\rho}}{(AE)_{o\rho}} = \frac{P_{A\lambda}\, l_{A\lambda}}{(AE)_{A\lambda}} = 40\ mm$$

	d mm	A_{mm^2}	E, GPa → N/mm^2	$AE_{(N)}$
op.	25	490,6	$105 \cdot 10^3$	$51,513 \cdot 10^6$
Al	60	2335,4	$70 \cdot 10^3$	$163,4 \cdot 10^6$

$$P_{op} \approx 40 \frac{(AE)_{op}}{l_{op}} \quad (l_{op} = 300 \text{ mm})$$

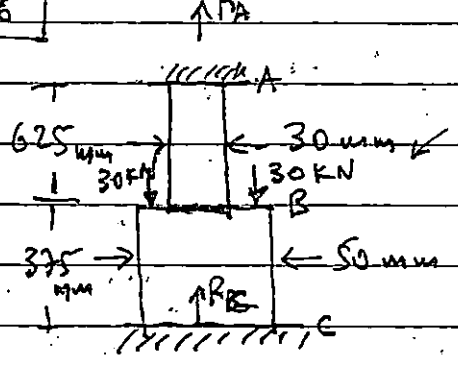
$$\approx 40 \cdot \frac{51,5 \cdot 10^6}{300} = 6,866 \cdot 10^6 \text{ N} = 6866 \text{ KN}$$

$$P_{Al} \approx 40 \frac{(AE)_{Al}}{300} = 40 \frac{163,4 \cdot 10^6}{300} = 21,8 \cdot 10^6 \text{ N}$$

$$P_{Al} = 21800 \text{ KN}$$

$$\therefore P_{\text{ολικό}} = \frac{6866}{21800 +} \approx \boxed{28666 \text{ KN}}$$
$$28666$$

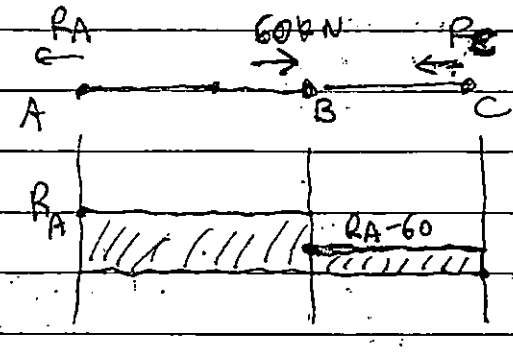
2.39/96



Πολυστυρένιο "E = 3.1 GPa"

Υπολ. (α) τις αντιδράσεις στα A, C.
(β) την αξονική τάση σε κάθε τμήμα της ράβδου

Απ.



(α) $R_A + R_C = 60$ KN, $\rightarrow R_C = 60 - R_A$.

Αξονική φόρτιση → εσωτ. = ε!

Ράβδος	d mm	A_{mm^2}	$(AE)_{(N)}$
(AB)	30	706,5	2190
(BC)	50	1962,5	6084

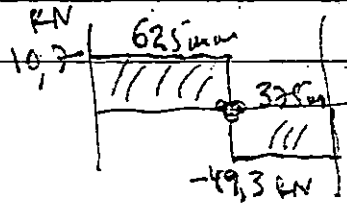
Πάκτωση / Πάκτωση → $\Delta l_{\text{ολικό}} = 0 \rightarrow \Delta l_{AB} + \Delta l_{BC} = 0 \rightarrow$

$$\frac{R_A (625)}{(AE)_{AB} 10^3} + \frac{(R_A - 60) \cdot 375}{(AE)_{BC} 10^3} = 0 \rightarrow \frac{R_A \cdot 625}{2190} + \frac{(R_A - 60) \cdot 375}{6084} = 0$$

$$\rightarrow R_A (0,285) + (R_A - 60)(0,062) = 0 \rightarrow R_A (0,347) = 3,72 \rightarrow$$

$$R_A = 10,7 \text{ KN}$$

$$R_C = 49,3 \text{ KN}$$



Καταστολή φορείου στη δεύτερη ράβδους.

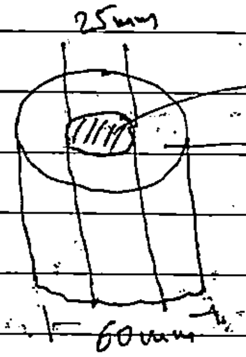
(b) ορθή τάση / ράβδο.

ράβδος	P(kN)	A mm²	σ=P/A (MPa)		Αστοχία	
AB	10,7	706,5	15,14	εφελκυστική	< 42,3	OK
BC	-49,3	1962,5	25,12	θλιπτική	< 69,2	OK.

δεν αστοχεί!!

σελ 891
Polystyrene → $\sigma_u^{εφ} = 55$ MPa → για γ=1,3 (συντ. ασφαλείας) $\sigma_{επ}^{εφ} = 42,3$ MPa.
πολυστυρένιο $\sigma_u^{θλ} = 90$ MPa → $\sigma_{επ}^{θλ} = 69,2$

2.47 / 98



	E GPa	α /°C	Δθ °C	αΔθ ·10^{-6}
Aλ.	70	23,6·10^{-6}	180	4248
Ορ.	105	20,9	180	3762

$\theta_{κεχ} = 15\,°C$, $\theta_{τελ} = 195\,°C$] $\Delta\theta = 180\,°C$

Υπολ. σ_{Al} =

Απ. $\Delta l_{Al} = l_{o,Al} \cdot \alpha_{Al} \cdot \Delta\theta \Longrightarrow \frac{\Delta l_{Al}}{l_{o,Al}} = \alpha_{Al} \cdot \Delta\theta = \epsilon_{Al}$

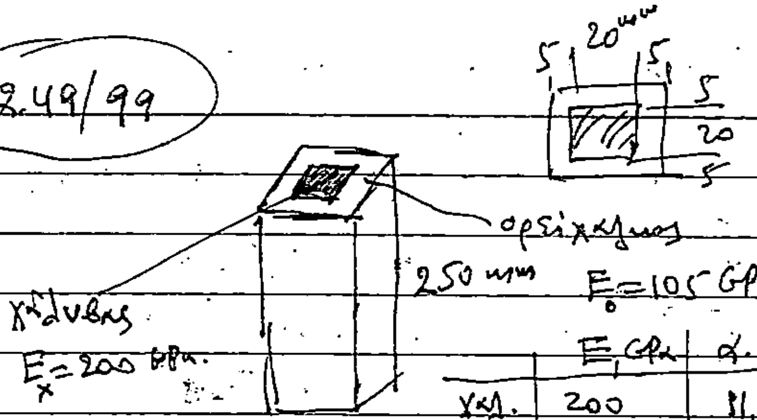
$\therefore \sigma_{Al} = E\,\epsilon_{Al} = 70 \cdot 10^3 \frac{N}{mm^2} \cdot 4248 \cdot 10^{-6} \cong \boxed{297 \text{ MPa}}$ εφελκυστική

σελ 890 Beer → $\sigma_{επ,Al}^{min} = \frac{\sigma_u}{\gamma} = \frac{110}{1,3} = 84,6$ MPa.

$\sigma_{επ,Al}^{max} = \frac{570}{1,3} = 438$ MPa

Σχόλ. Η αστοχία ή μη του αλουμινίου με την αύξηση της θερμοκρασίας κατά 180°C εξαρτάται από το είδος αλουμινίου που θα επιλέξει ο Μηχανικός.

2.49/99



	E, GPa	$\alpha \cdot 10^{-6}$/°C
χαλ.	200	11,7
ορ.	105	20,9

Υπολ. το Δϑ ώστε $\sigma_x \le 55$ MPa

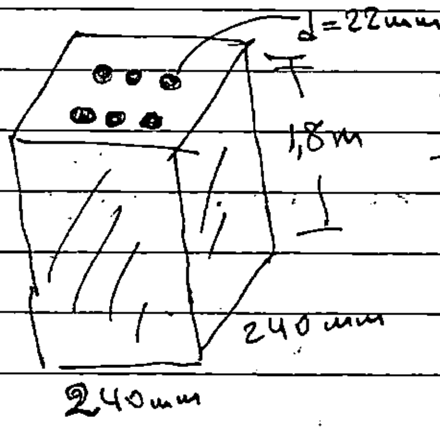
Απ.

$\Delta l_x = l_{0x} \alpha_x \Delta\vartheta \rightarrow \frac{\Delta l_x}{l_{0x}} = \boxed{\epsilon_x = \alpha_x \cdot \Delta\vartheta}$

$\sigma_x = E_x \epsilon_x = 200 \cdot 10^3 \frac{N}{mm^2} \cdot 11,7 \cdot 10^{-6} \cdot \Delta\vartheta = 2,34\ \Delta\vartheta$

$= 55$ MPa $\Rightarrow \Delta\vartheta = \frac{55}{2,34} = 23,5\ ^\circ C$!

2.50/100



	E, GPa	$\alpha \cdot 10^{-6}$/°C	Δϑ	$\alpha\Delta\vartheta \cdot 10^{-6}$
σκυρ.	25	9,9	35	346,5
χαλ.	200	11,7	35	409,5

Υπολ. τις ορθές τάσεις όταν $\Delta\vartheta = 35^\circ C$.

Απ.

σε 4 κάθε ράβδο. 10^3 MPa

$\epsilon_x = \alpha_x \Delta\vartheta = 346,5 (10^{-6}) \longrightarrow \sigma_x = 81900 \cdot 10^{-6}$ GPa $= 81,9$ MPa

$\epsilon_{σκ.} = \alpha_{σκ.} \Delta\vartheta = 409,5 (10^{-6}) \longrightarrow \sigma_{σκ.} = 8662 \cdot 10^{-6}$ GPa $= 8,66$ MPa

εφελκυστικές τάσεις

σελ. 890 $\rightarrow$ $\sigma^{σκυρ}_{u,θλ} = 40$ MPa (υψηλής αντοχής) ΥΑ
28 " (μετρίας ") ΜΑ

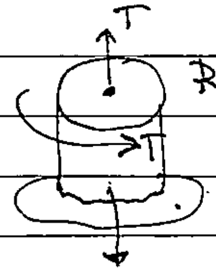
$\div 1,3 = \gamma$

$\sigma^{θλ}_{επ} = 30,76$ (ΥΑ) MPa,
$\sigma^{θλ}_{επ} = 21,5$ (ΜΑ)

$\sigma^{εφ}_{σκ} \simeq \frac{1}{10} \sigma^{θλ}_{σκ} = \Big\langle$ 3,07 MPa, 2,15

Σχόλιο. Το σκυρόδεμα δεν θα ραγίσει στη $\Delta\vartheta = 35^\circ C$!

3.1/178 $\tau_{max} = 70$ MPa. Υπολ: $T = ;$



Απ. (ροπή στρέψης)

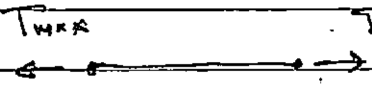
$\tau_{max} = \frac{T_{max} R}{J}$ → πολική ροπή αδράνειας

$J = \frac{\pi}{2} R^4 = \frac{\pi D^4}{32}$

$J = 164812 \; mm^4$

$T_{max} = 70 \frac{N}{mm^2} \frac{164812 \; mm^4}{18 \; mm} =$

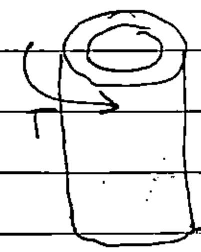
$T_{max} = 640936{,}8 \; N.mm = 640{,}9 \; N.m$

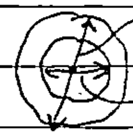


3.2/178 $T_{max} = 800 \; N.m \rightarrow \tau_{max} = \frac{800 . N . 10^3 mm . 18 mm}{164812 \; mm^4} =$

$\tau_{max} = 87{,}4$ MPa

3.3/178





90 mm = D = 2R

60 mm = d = 2r

$\tau_{max} = 45$ MPa

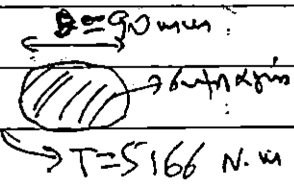
(α) $T = ;$

(β)

Απ. (α) $\tau_{max} = 45 \; MPa = \frac{T . R}{J} = \frac{T \cdot 45 \; mm}{\frac{\pi}{32}(90^4 - 60^4) \; mm^4} \Rightarrow$ (1 N/mm²)

$T = \frac{45 \frac{\pi}{32}(90^4 - 60^4)}{45} = 5166281 \; \frac{N.mm}{} \cong 5166 \; N.m$ ροπή στρέψης

(β)

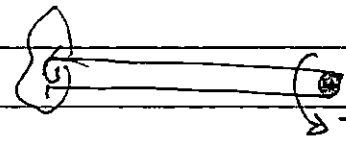


$\tau_{max} = \frac{5166 \cdot 10^3 \; N.mm}{\frac{\pi}{32} 90^4 \; mm^4} \cdot 90 \; mm = 72{,}2 \; MPa$

Σχόλ. Η max διατμ. τάση είναι μεγαλύτερη για συμπαγή κύλινδρο σε αντίθεση του κοίλου όταν οι T είναι τα ίδια!!

Άρα προτιμάμε κοίλους κυλίνδρους σε κατασκευές υπό στρέψη.

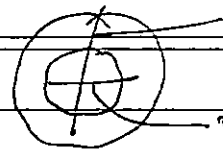
3.4 | 178



(α) Υπολ. το τ_{max} Απο: $\tau_{max} = \frac{T \cdot R}{J} = \frac{4,6 \cdot 10^3 \cdot 10^3 \text{ N mm} \cdot \frac{76}{2} \text{ mm}}{\frac{\pi}{32} 76^4}$

$\tau_{max} \approx 53,4$ MPa.

(β)



$$\tau_{max} = \frac{4,6 \cdot 10^6 \text{ N.mm} \cdot \frac{76}{2} \text{ mm}}{\frac{\pi}{32}(76^4 - 24^4) \text{ mm}^4} \approx 53,9 \text{ MPa}$$

→38

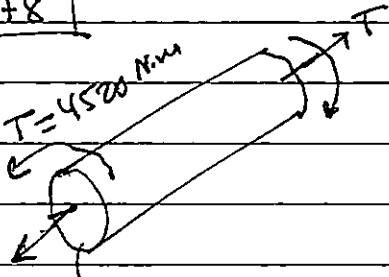
$\frac{\pi}{32}(76^4 - 24^4)$

33362176 → 331776

$333 \cdot 10^6$ >> $0,33\ 10^6$

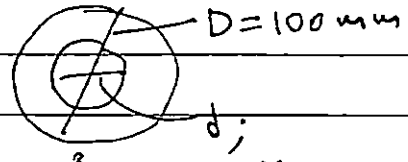
Περίπου ίδια διαφορά, τάση

3.5 | 178



(α) $\tau_{max} = \frac{4520 \cdot 10^3 \text{ N mm} \cdot \frac{75}{2} \text{ mm}}{\frac{\pi}{32} 75^4 \text{ mm}^4} = 54,6 \text{ N/mm}^2$ MPa

(β) d =;

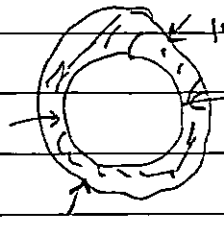


$$\tau_{max} = 54,6 \text{ MPa} = \frac{4520 \cdot 10^3 \text{ N.mm} \cdot \frac{100}{2} \text{ mm}}{\frac{\pi}{32}(100^4 - d^4) \text{ mm}^4} \Rightarrow$$

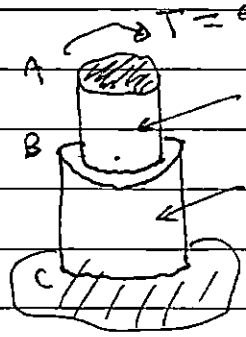
$$100^4 - d^4 = \frac{4520 \cdot 10^3 \cdot 50}{\frac{\pi}{32}(54,6)} = 42187500$$

10^8

$d = (57812500)^{1/4} = 87,2$ mm



3.16/181 $\tau_{max,X} = 100$ MPa $\tau_{max,O} = 6$ MPa.



$d_x =$; $d_o =$;

AB

$$\tau_{max,X} = 100 \frac{N}{mm^2} = \frac{T \cdot \frac{d_x}{2}}{\frac{\pi}{32} d_x^4} \Rightarrow$$

$$\frac{N}{mm^2} 100 = \frac{900 \cdot 10^3 \ N.mm}{\frac{\pi}{16} d_x^3 \ mm^3} \Rightarrow d_x^3 = \frac{9000}{\frac{\pi}{16}} = 45859{,}8$$

$$d_x = 35{,}79 \text{ mm} \quad \checkmark$$

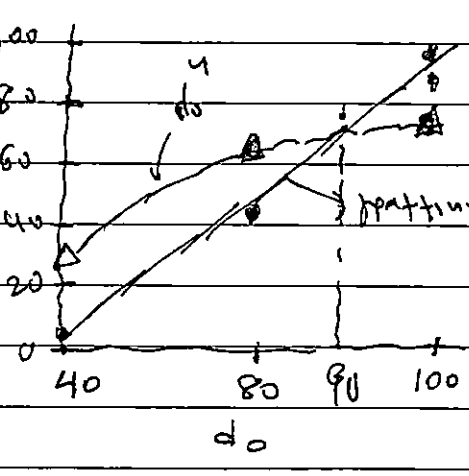
BC

$$\frac{N}{mm^2} 6 = \frac{900 \cdot 10^3 \ N.mm \cdot \frac{d_o}{2} \ mm}{\frac{\pi}{32}(d_o^4 - 35{,}79^4)} \Rightarrow d_o^4 - 35{,}79^4 = \frac{900 \cdot 10^3 \cdot 16}{\pi(6)} d_o \Rightarrow$$

$$d_o^4 - 764331 d_o - 1642804 = 0 \qquad d_o \text{ (mm)}$$

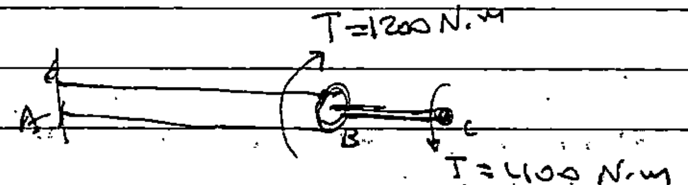
Αντί για να βρούμε τη λύση της εξίσωσης 4ου βαθμού, υπολ. το δεξιό μέλος για διάφορες τιμές του d_o.

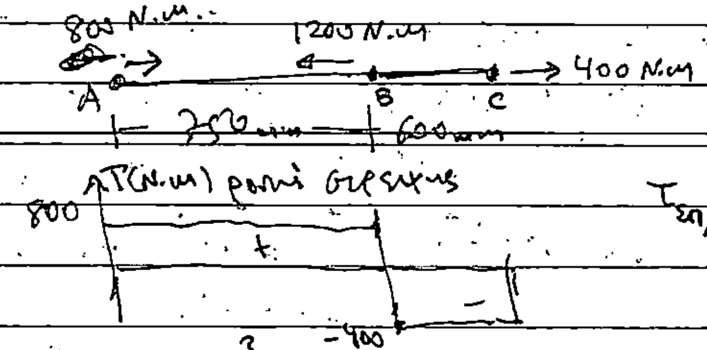
d_o (mm)	ΔM	ΔM
100	100,000,000	76,449,836
80	40,960,000	62,789,081
70	24,010,000	53,673,116
110	146,410,000	842,405,596
40	2,560,000	32,215,841
37	1,874,161	28,443,820
90	$65{,}6 \cdot 10^6$	70,43



→ Οπότε $d_o \simeq 88$ mm

3.17/181 $\tau_{\epsilon\pi,o\lambda} = 55\ MPa$





$d_{AB} = ;$ $d_{BC} = ;$

$$\tau_{\epsilon\pi,o} = 55\ \frac{N}{mm^2} = \frac{T_{AB} \cdot d_{AB}/2}{\frac{\pi}{32} d_{AB}^4}, \quad T_{AB} = 800 \cdot 10^3\ N.mm$$

$$55 = \frac{16 \cdot 800 \cdot 10^3}{\pi d_{AB}^3} \rightarrow$$

$$d_{AB}^3 = \frac{16(800) \cdot 10^3}{3{,}14(55)} = d_{AB}^3 = 74117 \Rightarrow$$

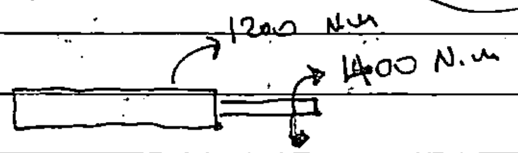
$$\boxed{d_{AB} \approx 42\ mm}$$

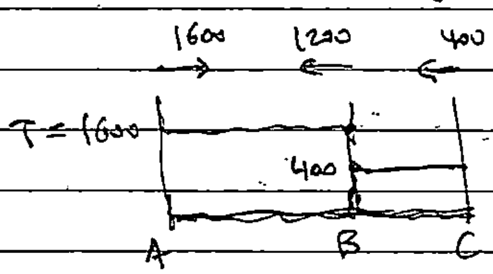
$$55 = \frac{T_{BC} \cdot d_{BC}/2}{\frac{\pi}{32} d_{BC}^4} \Longrightarrow \quad T_{BC} = 400 \cdot 10^3$$

$$d_{BC}^3 = \frac{16(400)\ 10^3}{\pi(55)} = 37058 \rightarrow$$

$$\boxed{d_{BC} = 33{,}34\ mm}$$

3.18/181





$$d_{AB}^3 = \frac{16(1600) \cdot 10^3}{3{,}14(55)} = 148234 \rightarrow \boxed{d_{AB} = 52{,}9\ mm}$$

$$\boxed{d_{BC} = 33{,}34\ mm}$$

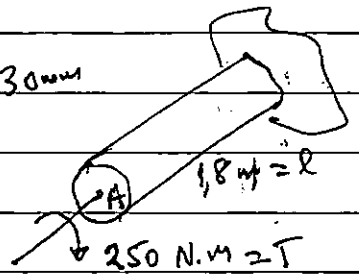
Συμπ.

	d_{AB} (mm)	d_{BC} (mm)
1200, 400	42	33,34
1200, 400	52,9	33,34

∴ Διαφορές ⇒ αύξηση στο d_{AB}

3.33 | 195 $G = 77\ GPa$. $\phi_A = ;$

(α)

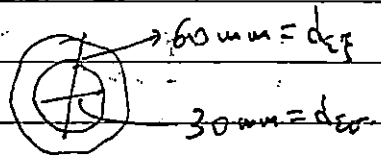


$$\phi_A = \frac{T \cdot l}{J G} = \frac{250 \cdot 10^3\ N.mm \cdot 1{,}8 \cdot 10^3\ mm}{\left(\frac{\pi}{32} \cdot 30^4\ mm^4 \cdot 77 \cdot 10^3\ \frac{N}{mm^2}\right)} =$$

$$= 79481.$$

$$\phi_A = 0{,}0735\ RAD \xrightarrow{\times \frac{180^\circ}{\pi}} \boxed{4{,}2^\circ}$$

(β)



$$J = \frac{\pi}{32}\left(60^4 - 30^4\right) = 1{,}192{,}218\ mm^4$$

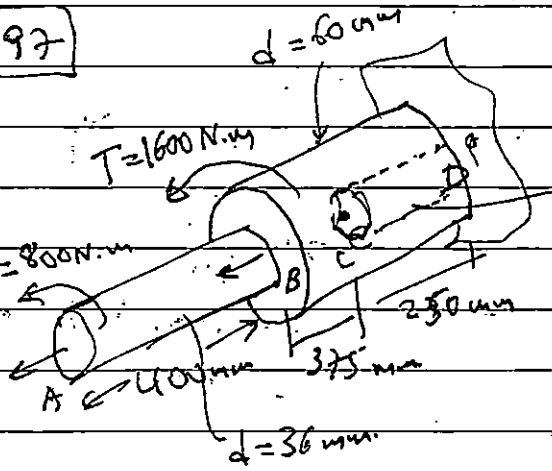
$$\phi_A = \frac{250 \cdot 10^3\ N.mm \cdot 1{,}8 \cdot 10^3\ mm}{1{,}192{,}218\ mm^4 \cdot (77) \cdot 10^3\ \frac{N}{mm^2}} = 0{,}0049$$

$\times \frac{180}{\pi}$

$0{,}28^\circ$

Σχόλιο. Η γωνία στρέψης στον ιδίο δίσκο είναι πολύ μικρότερη.

3.38 | 197

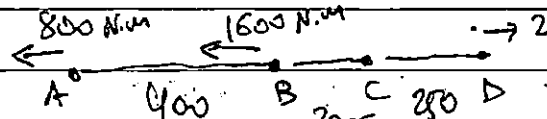


Δίνεται $G = 27\ GPa = 27 \cdot 10^3\ \frac{N}{mm^2}$

ισιός έως $d = 40$ mm.

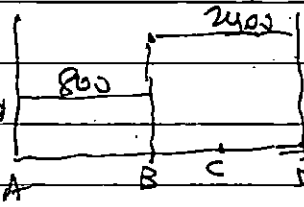
Υπολογίστε τη γωνία στρέψης ϕ_A

Δ.Ο.



$$\phi_A = \frac{T_{AB} l_{AB}}{J_{AB} G} + \frac{T_{BC} l_{BC}}{J_{BC} G} + \frac{T_{CD} l_{CD}}{J_{CD} G}$$

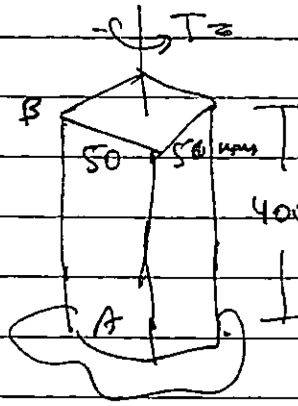
	d (mm)	J mm^4	T (N.mm)	l mm	ϕ_{ij}
AB	36	164812	$800 \cdot (10^3)$	400	0,072
BC	60	1271700	$2400 (10^3)$	375	0,026
CD	60/40	1,020,500	$2400 (10^3)$	250	0,022



$\phi_A = 0{,}120$ rad

$1{,}25^\circ$

3.125 | 238



$\tau_{επ} = 50$ MPa

$G = 39$ GPa. "ορ. ψυχρής έλασης"

Υπολ. το T κ' ϕ_B για

50x50 δοκό κ'

35x70 mm "

Απ. Διατομή σελ.

$$\tau_{επ} = \frac{T}{C_1 a b^2}$$

↓ στενότερη πλευρά

για $\frac{a}{b} = 1 \rightarrow C_1 = 0{,}208$

$\frac{a}{b} = \frac{70}{35} = 2 \rightarrow C_1 = 0{,}246$.

Πιν. 3.1 σελ 231

$$50 \frac{N}{mm^2} = \frac{T\ (N.mm)}{0{,}208 \cdot 50 \cdot 50^2\ mm^3} \Rightarrow T = 1300\,000\ N.mm$$

$$T_{\square 50 \times 50} = 1300\ N.m$$

$$50 = \frac{T}{0{,}246 \cdot 70 \cdot 35^2} \rightarrow T_{\square 35 \times 70} = 1054{,}7\ N.m$$

Γωνία στρέψης φ.

για 50x50 $\frac{a}{b} = 1$ $C_2 = 0{,}1406$

70x35 $\frac{a}{b} = 2$ $C_2 = 0{,}229$

$$\phi = \frac{T.L}{C_2 a b^3 \cdot G}$$

3.126

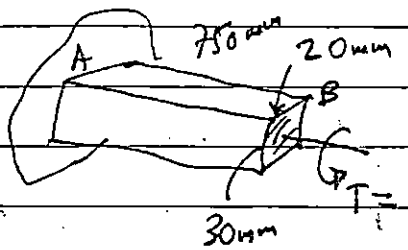
$$\phi_{\square 50 \times 50} = \frac{800 \cdot 10^3 \cdot 400}{0{,}1406 \cdot 50\ 50^3 \cdot 39 \cdot 10^3} = 0{,}00934\ RAD \longrightarrow 0{,}53^\circ$$

$$\phi_{70 \times 35} = \frac{800 \cdot 10^3 \cdot 400}{0{,}229 \cdot 70 \cdot 35^3 \cdot 39 \cdot 10^3} = 0{,}012 \longrightarrow 0{,}038^\circ$$

Διατομή	T (N.m)	ϕ°
50x50	1300	0,53
70x35	1055	0,038

→ επιλέγουμε αυτή τη διατομή!! διότι παρουσιάζει τιμή γωνία στρέψης

3.128/238



$G = 75.10^3 \ N/mm^2$

$\phi_B = 2^0 = 0,035 \ rad$

Yêu cầu: τ_{max}

Ta có $\tau_{max} = \dfrac{T.\ell}{c_1 . a b^2}$ $\left(\dfrac{N.mm^2}{mm^4} = N/mm^2 = MPa\right)$

$\phi_B = \dfrac{T.\ell}{c_2 . a b^3 G} = 0,035$

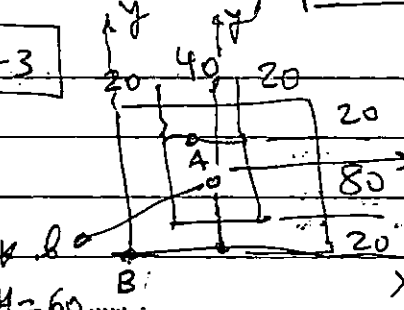
$\dfrac{a}{b} = \dfrac{30}{20} = 1,5 \rightarrow c_2 = 0,1958$

$0,035 = \dfrac{750 \ T(N.mm)}{0,1958 \ 30 \ 20^3 . 75.10^3} \Rightarrow \rightarrow c_1 = 0,231$

$T = 16447,2.10^3 \ N.mm$

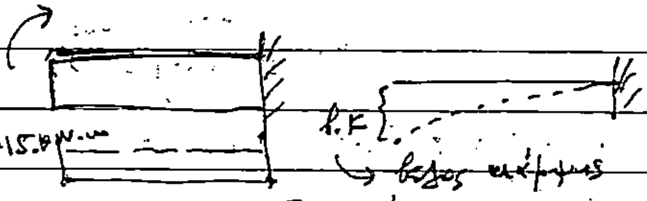
$\tau_{max} = \dfrac{16447,2.10^3}{0,231 . 30 . 20^2} = \boxed{59,3 \ MPa}$

4.1/273



$y_{κβ} = 60$ mm

$M = 15$ kN.m $= 15 \cdot 10^6$ N.mm



Υπολ. των $σ_A$ & $σ_B$.

Απ.:

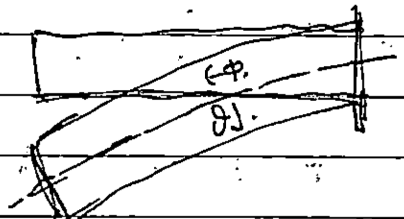
$$σ_A = \frac{M_A \cdot y_A}{I_{κβ}} = \frac{15 \cdot 10^6 \text{ N.mm} \cdot 20 \text{(mm)}}{11\,520\,000} = \frac{15 \cdot (20) \cdot 10^6}{1152 (10^4)} = 26 \text{ MPa}$$ εφελκ./ισ.

$$I = \frac{1}{12} 80 \cdot 120^3 = 11{,}520{,}000 \text{ mm}^4$$

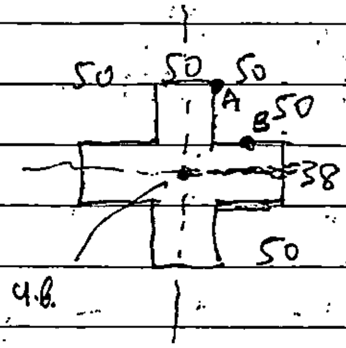
$$σ_B = \frac{M_B}{I_{κβ}} y_B = \frac{15 \cdot 10^6 \cdot (60)}{1152 \cdot 10^4} = 3 σ_A = 78 \text{ MPa}$$ → θλιπτική τάση.

→ Αρχικά πρόσημα και υπολ. το μέτρο βάσης: λόγω συμμετρίας

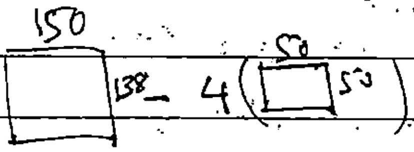
$y_{κβ} = 60$ mm



4.2/273



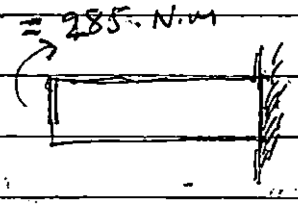
Ροπή αδράνειας επιφάνειας (σταυρού).



$$I_{κβ} = \frac{1}{12} 150 \cdot 138^3 - 4\left(\frac{1}{12} 50 \cdot 50^3\right) =$$

$$= 32{,}850{,}900 - 2{,}083{,}333 = 30{,}76 \cdot 10^6 \text{ mm}^4$$

$T = 285$ N.m



$y_A = 50 + 19$ mm $= 69$ mm

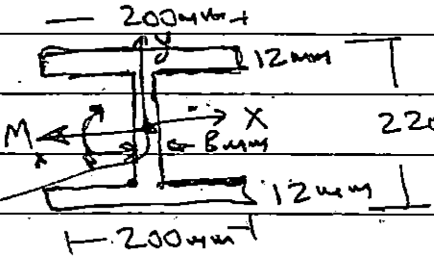
$y_B = 19$ mm

$$σ_A = \frac{M_A}{I} y_A = \frac{285 \cdot 10^3 \text{ N.mm} (69) \text{ mm}}{30{,}76 \cdot 10^6 \text{ mm}^4} = 0{,}639 \text{ MPa}$$

$$σ_B = \frac{285 \cdot 10^3 \cdot (19)}{30{,}76 \cdot 10^6} = 0{,}176 \text{ MPa}$$

4.3 | 273 $\sigma_{επ} = 155\ MPa$ $M_{max} = ;$

W-δοκός



Απ.

$\sigma_{επ} = 155 = \frac{M_{max}}{I} y_{max}$ κ.β. $\{ x = 0,\ y = 110mm$

110 mm (απόσταση κ.β. από την άνω επιφάνεια του Ι προφίλ)

$I_{κβ} = $ 220 [200] $- 2$ [ ] $220 - 24 = 196 =$

$\frac{200-8}{2} = \frac{192}{2} = 96$

$= \frac{1}{12} 200 \cdot 200^3 - 2 \frac{1}{12} 96 \cdot 196^3 = 133,333,333 - 120,472,576 =$

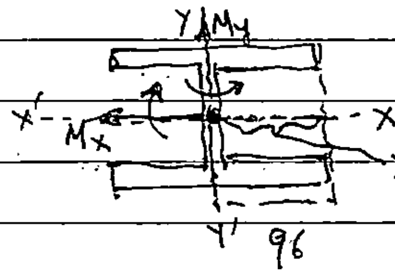
$= \boxed{12,8 \cdot 10^6\ mm^4}$

$\therefore\ 155\ N/mm^2 = \frac{M_{max}(110\ mm)}{12,8 \cdot 10^6\ mm^4} \Rightarrow M_{max} = 18,12 \cdot 10^6\ N.mm =$

$\boxed{M_{max,x} = 18,12\ kN.m}$

4.4 | 273

My - ροπή κάμψης γύρω από τον y-άξονα.

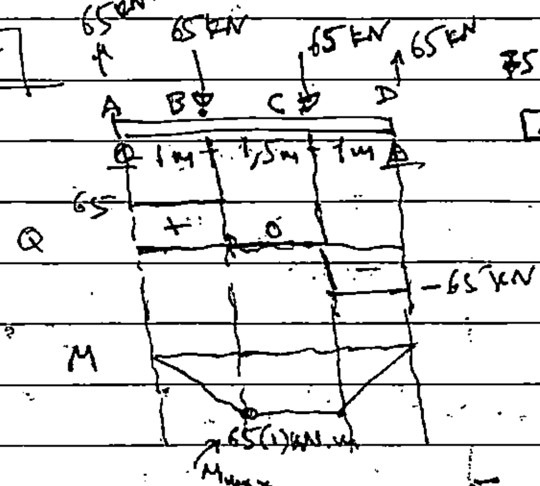


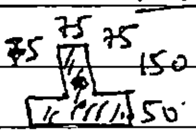
$\sigma = \frac{M_y}{I_{κβ}} \cdot x_{max} \Rightarrow M_y = 155 \cdot \frac{12,8 \cdot 10^6}{96} =$

$= 20,6 \cdot 10^6\ N.mm$

$\boxed{M_x^{max} = 20,6\ kN.m}$

4.9 | 275





Υπολ.: $\sigma^{εφ}_{max}$, $\sigma^{θλ}_{max}$

Απ.

Στατική: $130 = A + D$

$65(1) + 65(2,5) = D(3,5)$

$D = 65\ kN$

$A = 65\ kN$

N-Q-M διαγράμματα

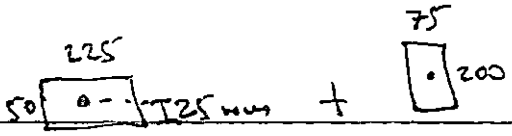
65kN.m

$\sigma_{max} = \frac{M_{max}}{I_{κβ}} \cdot y_{max}$

Υπολ. το κ.β.

$x_{κβ}=0$

$y_{κβ}=$



	A	$\bar{Y}$	$A\bar{Y}$
1	225 x 50	25	281250
2	200 x 75	125	1875000
	26250		2156250

$y_{κβ} = 82,14$ mm

Υπολ. το $I_{κβ}$.

$y = 200 - 82,14 = 117,86$ mm
άνω επιφ.

$y_{κάτω\,επιφ.} = 82,14$ mm

$I_{κβ} = \frac{1}{12}\,225\cdot 50^3 + \frac{1}{12}\,75\cdot 200^3 = 2{,}343{,}750 + 50{,}000{,}000 = 52{,}3\cdot 10^6\ mm^4$

Άρα: $\sigma^{BC}_{max,άνω} = \frac{65\cdot 10^6 \cdot 117,86}{52,3\cdot 10^6} = 146,5$ MPa θλιπτικό!

$\sigma^{BC}_{max,κάτω} = \frac{65\cdot 10^6 \cdot 82,14}{52,3\cdot 10^6} = 102$ MPa εφελκυστικό!

Σελ. 890: Εάν το υλικό είναι από χυτοσίδηρο (χημ. σύνθεσης 4,5% C) ASTM A-48 → $\sigma_{u,εφ} = 170$ MPa, $\sigma_{u,θλ} = 655$ MPa $\div \gamma = 1,3$ → $\sigma_{επ,εφ} = 130$, $\sigma_{επ,θλ} = 503,8$ MPa
($E = 165$ GPa)

$\sigma_{max,άνω} = 146,5 < 503,8$ MPa
$\sigma_{max,κάτω} = 102 < 130$ MPa
δεν χρειάζεται γι' αυτό την ανάλυση.

Συμπ: (1) Η δοκός αντέχει το φορτίο 65 kN εφαρμοζόμενο του κέντρου, διατμητικής περιοχ. ⊥ με τις ανωτέρω διαστάσεις.

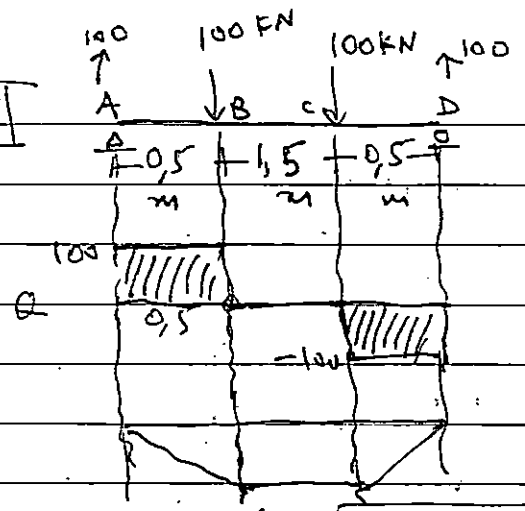
(2) Εάν το υλικό είναι από ξύλο

$\sigma_{u,εφ} = 100$ MPa $\xrightarrow{\gamma=3}$ 33,3 MPa $= \sigma_{επ,εφ}$

Σελ 891 $\sigma_{u,θλ} = 50$ → 17 MPa $= \sigma_{επ,θλ}$

Η δοκός δεν αντέχει το φορτίο, θα σπάσει!

4.10 | 275



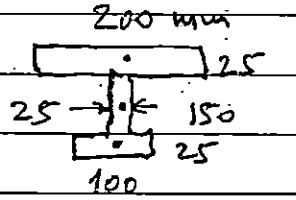
Στατική: $A + D = 200$

$100(0,5) + 100(2) = D(3,5)$

$D = 100$ KN

$A = 100$

$100(0,5) = 50$ KN.m $= M_{max}$



Προσδιορισμός του $I_{κβ.}$

$$I = \frac{1}{12}100\,25^3 + \frac{1}{12}25\,150^3 + \frac{1}{12}200\,25^3 + \text{[100 × 25]} + \text{[25 × 150]} + \text{[200 × 25]}$$

$$\simeq \frac{1}{12}300(25)^3 + \frac{1}{12}25(150)^3 =$$

$$= 390,625 + 7,031,250 = \boxed{7,4 \times (10^6)\ mm^4}$$

Προσδ. θέσ. κ.β.

$\bar{x} = 0$

		A_i, mm²	$\bar{y}_i$, mm	$A_i\bar{y}_i$
1	200 × 25	$200(25) = 5000$	$25 + 150 + \frac{25}{2} = 187,5$	937500
2	150 × 25	$150(25) = 3750$	$25 + \frac{150}{2} = 100$	375000
3	100 × 25	$100(25) = 2500$	$\frac{25}{2} = 12,5$	31250
		11250		1343750

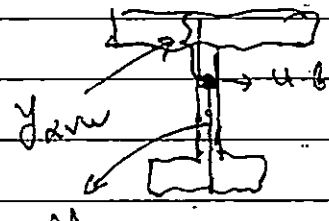
$$\boxed{\bar{y}_{κβ} = 119,44\ mm}$$

$y_{κκω} = 300 - 119,44 = 180,56$ mm

$y_{κκω} = 119,44$ mm

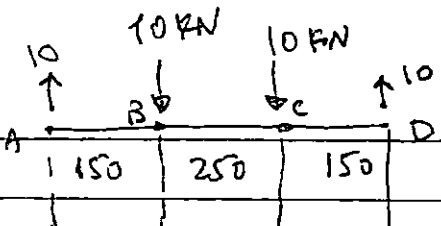
Ορθή τάση:

$$\sigma_{κω} \simeq \frac{M_{max}}{I} \cdot y_{κω} = \frac{50 \cdot 10^6}{7,4\ 10^6} 180,56 = 1220\ MPa.$$



$$\sigma_{κκτω} \simeq \frac{50\ 10^6}{7,4\ 10^6} 119,44 = 807\ MPa.$$

4.11 / 275

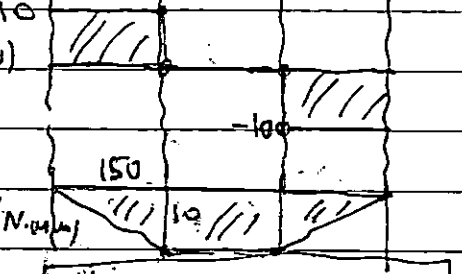


Στατική: ως άνω

$A = D = 10\ kN \uparrow$

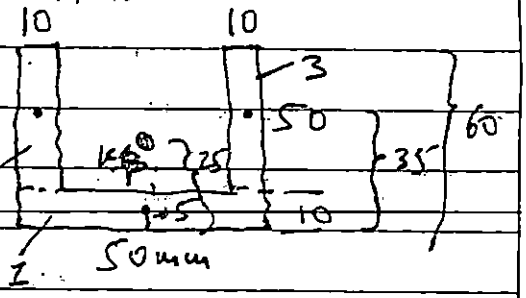
ΝQΜ

Διαγράμματα:

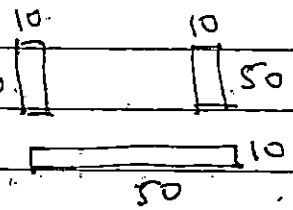


$M_{max} = 150 \cdot 10 = 1500\ N.mm$

Διατομή:



Προσδ. του $I_{κ.β.}$



$$I_{κβ} = \frac{1}{12} 50^3 10 + \frac{1}{12} 50^3 10 + \frac{1}{12} 50 \cdot 10^3$$

$$= 208{,}333 + 4166 = 212{,}500$$

$$I_{κβ} = 212{,}5 \cdot (10^3)\ mm^3$$

Προσδ. του κ.β.

	A_i	$\bar{y}_i$	$A_i \bar{y}_i$
1	$50 \times 10 = 500$	5	2500
2	$50 \times 10 = 500$	35	17500
3	$50 \times 10 = 500$	35	17500
	1500		37500

$$\bar{y}_{κβ} = \frac{37500}{1500} = 25\ mm.$$

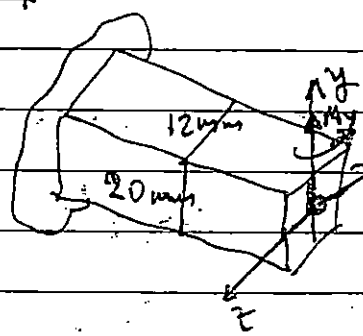
$y_{άνω} = 60 - 25 = 35\ mm$

$y_{κάτω} = 25\ mm.$

Ορθές Τάσεις: $\sigma_{max}^{άνω} = \frac{M_{max}}{I_{κ.β.}} y_{άνω} = \frac{1500}{212{,}5 \cdot 10^3} \cdot 35 = 0{,}247\ MPa$ θλιπτική!

$\sigma_{max}^{κάτω} = \frac{M_{max}}{I_{κβ}} y_{κάτω} = \frac{1500}{212{,}5 \cdot 10^3} \cdot 25 = 0{,}176\ MPa$ εφελκυστική!

4.24 / 278



(α) $\sigma_{max,z}$; για $M_z = 60\ N.m$

(β) $\sigma_{max,y}$; για $M_y = 60\ N.m$

$E = 200\ GPa$

Η κυρίαρχη υπόθεση;

Απ. (α) $\sigma_{max,z} = \frac{M_z}{I_z} \cdot y_{max} = \frac{60 \cdot 10^3\ N.mm}{\frac{1}{12} 12 (20)^3} \cdot \frac{20}{2} = 0{,}75\ MPa$

$$\rho^{-1} = \frac{M_z}{E I_z} = \frac{60 \cdot 10^3 \text{ N.mm}}{200 \cdot 10^3 \frac{\text{N}}{\text{mm}^2} \cdot \frac{1}{12} 12 \cdot 20^3 \text{ mm}^4} = 3{,}75 \cdot 10^{-5} \text{ mm}^{-1}$$

$$\boxed{\rho_1 \approx 26{,}6 \text{ m}} \qquad I_z = 8000 \text{ mm}^4$$

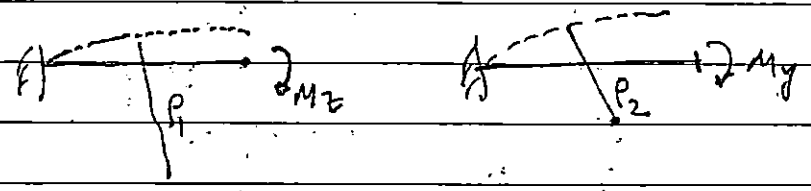
(β) $M_y = 60 \text{ N.m} \rightarrow \sigma_{max,y} = \frac{M_y}{I_y}(t/2) = \frac{60 \cdot 10^3}{\frac{1}{12}(12)^3 \cdot 20} \cdot (12/2) =$

$$= \frac{60 \cdot 10^3}{(12)^1 \, 2{,}0} = \frac{1000}{4} = \boxed{250 \text{ MPa}} \qquad I_y = \frac{1}{12} 12^3 \cdot 20 = 2880 \text{ mm}^4$$

γ'

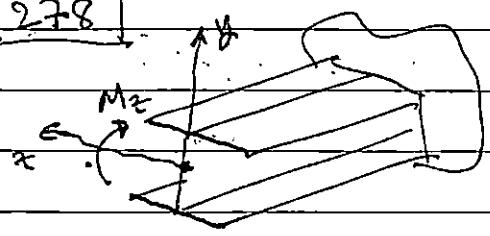
$$\rho^{-1} = \frac{M_y}{E I_y} = \frac{60 \cdot 10^3}{200 \cdot 10^3} \frac{1}{\frac{1}{12} 12^3 \cdot 20} = \frac{6}{(20)^2 \cdot 12^2} \text{ mm}^{-1} \rightarrow$$

$$\boxed{\rho = 9{,}6 \text{ m}}$$



Σημείωση $(\rho \sim I) \Rightarrow$ για ίδιο υλικό Ι έχουμε ίδιες ακτίνες καμπυλότητας

4.26 \| 278



$M_z = 24 \text{ kN.m} = 24 \cdot (10^6) \text{ N.mm}$

W 200x46,1 (Διάταξη)

(α) $\sigma = \frac{M_z}{I_z}(y/2)$

(β) $\rho^{-1} = \frac{M_z}{E I_z} = \frac{24 \cdot 10^6 \text{ N.mm}}{200 \cdot 10^3 \cdot 458 \cdot 10^6 \frac{\text{N}}{\text{mm}^2} \text{ mm}^4}$ για W200x46,1 $\rightarrow I_z = 458 \cdot 10^6 \text{ mm}^4$ σελ. 897 $(x \rightarrow z)$

$$\boxed{\rho_z \approx 381 \text{ m}} \checkmark$$

$y = d = 203 \text{ mm}$

$$\therefore \sigma = \frac{24 \cdot 10^6 \text{ N.mm}}{458 \cdot 10^6 \text{ mm}^4} \cdot \frac{203}{2} \text{ mm} =$$

$$\boxed{\sigma_{max} = 53{,}18 \text{ MPa}}$$

(β) $M_y = 24\,(10^6)$ N.mm.

$I_y = 15{,}4\,(10^6)$ mm^4

$$\therefore \sigma = \frac{M_y}{I_y}\cdot(h/2) \simeq \frac{24\cdot 10^6}{15{,}4\,10^6}\cdot\frac{203}{2} = \boxed{158\ MPa}$$

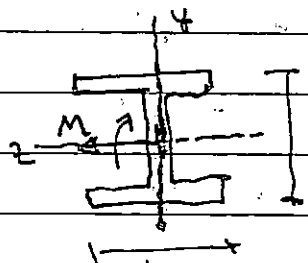
$$\rho_y = \frac{E I_y}{M} \simeq \frac{200\cdot 10^3 \cdot 15{,}4\,10^6}{24\cdot 10^6}\,(mm) = 128{,}3\cdot 10^3\ mm \quad (10^{-3}\ km)$$

$$\boxed{\rho_y = 128{,}3\ km}\ V$$

4.31/279 W200 x 31,3

$M = 45$ kN·m $E = 200$ GPa $\nu = 0{,}29$.

(α)



GSA. 897

$I_z = 31{,}3\cdot 10^6$ mm^4

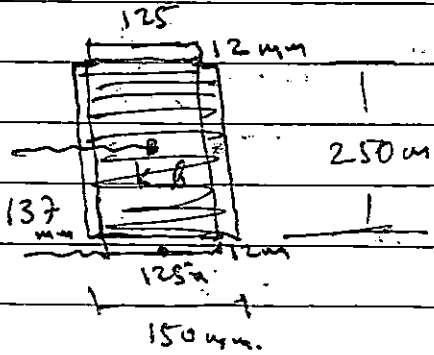
$$\rho = \frac{E I_z}{M_z} = \frac{200\cdot 10^3\ \frac{N}{mm^2}\cdot 31{,}3\cdot 10^6\ mm^4}{45\cdot 10^6\ N\cdot mm} =$$

$$\boxed{\rho_1 = 139{,}1\ m}$$

(β) Εγκάρσια Διάταξη $y \rightarrow I_y = 4{,}07\cdot 10^6$ mm^4

$$\rho_2 = \frac{E I_y}{M_z} = \frac{200\cdot 10^3\cdot 4{,}07\cdot 10^6}{45\cdot 10^6} = \boxed{18\ m}$$

4.37/289



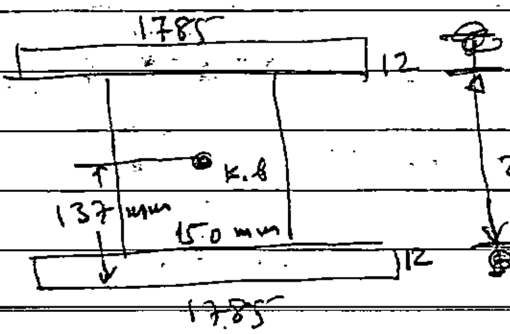
$\bar{x} = 0$. $\bar{y}_{κβ} = 137$ mm

$$\sigma = \frac{M}{I_{x_{ουδ}}}\cdot y_{max} \le \sigma_{επ}^{χαλ} = 150\ MPa.$$

$$n \simeq \frac{R_x}{E_=} = \frac{200}{14} = 14{,}28\ \text{"μικροκυματικός"}.$$

Πολ/ζουμε τη διάσταση 125mm του χαλύβδινου ελάσματος x $n = 14{,}28$

$= 1785$ mm



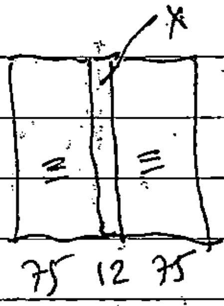
$$I_{κβ} = \frac{1}{12}(1785)\,12^3 \times 2 + \frac{1}{12}\,150\cdot(250)^3 \approx 195{,}5\cdot 10^6\ (mm^4)$$

257040 195312500

$$\sigma_{ξύλο} = \frac{M}{I}\cdot(125+12) \approx 14\ MPa \rightarrow M = \frac{14\cdot 195{,}5\cdot 10^6}{137}\ N.mm \approx 20\ kN.m$$

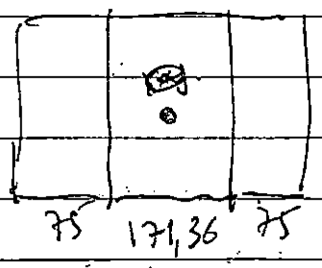
$$\sigma_{χαλ} = n\frac{M}{I}(137) = 14{,}28\cdot\frac{20\cdot 10^6}{195{,}5\cdot 10^6}\,137 \approx 200\ MPa$$

4.38/290



$n = 14{,}28$ (συντ. μετασχ. διατομής)

$12\,n = 171{,}36$



$$I = \frac{1}{12}(75)\,250^3 \times 2 + \frac{1}{12}\,171{,}36\,250^3$$

$$\approx \frac{(150+171{,}36)}{12}\,250^3 =$$

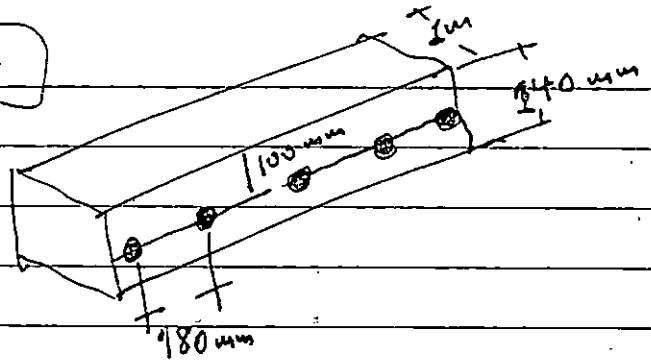
$$I = 418\cdot 10^6\ mm^4$$

$$\sigma = \frac{M}{I}\left(\frac{h}{2}\right) \approx 14\ MPa \rightarrow M \approx \frac{14\times 418\cdot 10^6}{125}\ N.mm$$

(h/2 → 125; N.mm → kN.m)

$$M = 46{,}8\ kN.m$$

4.47/291



$d_x = 16$ mm

$E_{ελ} = 20$ GPa

$E_x = 200$ GPa

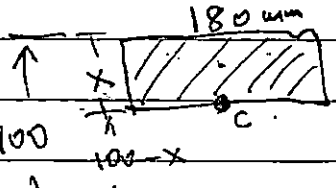
$n = \frac{200}{20} = 10.$

$\sigma_{εφ,ελ} = 9$ MPa , $\sigma_{επ,x} = 120$ MPa.

$A_x = \frac{\pi}{4} d_5^2 \approx 200,96$ mm^2

Ισορροπία ροπών ως προς το c

$180 \cdot x \cdot \frac{x}{2} = 200,96 \cdot (100 - x)$



$90x^2 + 200,96x - 200960 = 0$

$nA_5 \approx 2009,6$ mm^2

$x^2 + 22,33x - 2232,8 = 0$

$I_{u.β} = \frac{1}{12} 180 x^3 + 2009,6 (100 - x)^{62,6}$

$x = \frac{-22,33 \pm \sqrt{22,33^2 + 4(2232,8)}}{2}$

$= 1,78 \cdot 10^6 + 1,25 \cdot 10^6 = 2,03 (10^6)$ mm^4

$x = \frac{-22,33 \pm 97,11}{2} = 37,4$ mm

Επιτρ. ροπή ως προς εφελκυσμό

$\sigma = \frac{nM}{I} \cdot y \xrightarrow{x} 150 \Rightarrow M = \frac{150 \cdot 2,03 \cdot 10^6}{37,4}$ N.mm $\rightarrow$ 1 kN.m

$M = 8,11$ kN.m $\quad$ $M = 9488$ kN.m
εφ.

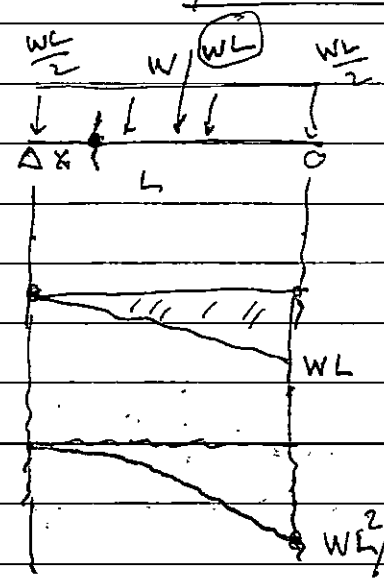
Επιτρ. ροπή ως προς x<1mm

$\sigma = \frac{M \cdot n \cdot y_x}{I} \xrightarrow{10} \xrightarrow{62,6} 150 \rightarrow M = \frac{150 (2,03) (10^6)}{10 (62,6)}$ kN.m

$M_{χαλ.} = 9,486$ kN.m

ΚΕΦ. 5°

5-1 / 379



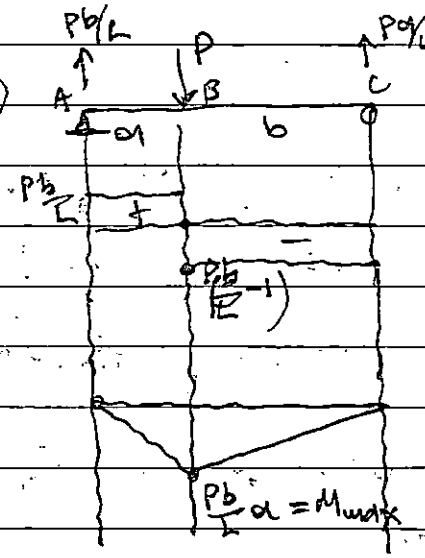
$q = (wx)$

$V = w \cdot x$ ευθεία

$M = wx \frac{x}{2} = w\frac{x^2}{2}$ παραβολή

$\left.\frac{dM}{dx}\right|_{x=0} = 0$. (η παραβολή αρχίζει από $x=0$ ⊥ στον ικτινόμενο άξονα.

5.2 / 379



$a + b = L$.

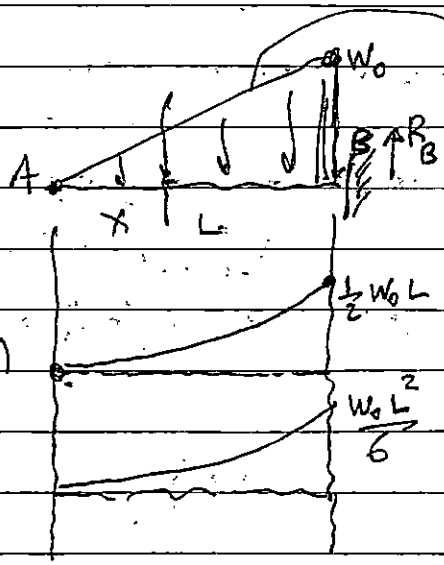
Στατική $A + C = P$

$Pa = C \cdot (a+C) \Rightarrow C = P\frac{a}{L}$

$A = \frac{Pb}{L}$

$P\left(\frac{b}{L} - 1\right) \cdot b = \frac{Pab}{L}$

5.3 / 379



$W(x) = Ax + B$ ευθεία!

$x = 0 \rightarrow w = 0 = B$

$x = L \rightarrow w = W_0 = A \cdot L \rightarrow A = \frac{W_0}{L}$

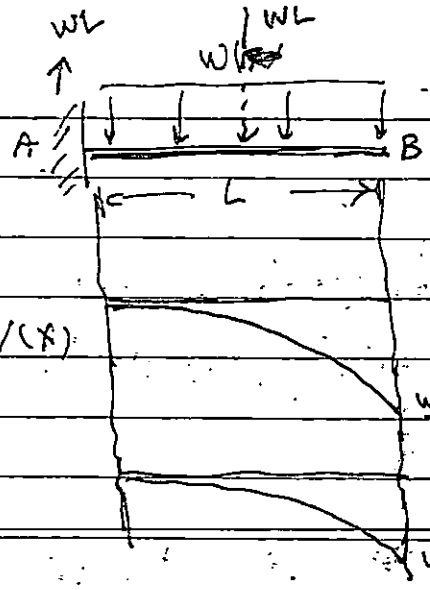
$$W(x) = \frac{W_0}{L} \cdot x$$

$V(x) = \int_0^L \frac{W_0}{L} x \, dx = \frac{W_0 L^2}{2L} = \frac{W_0 L}{2}$

$V(x) = \frac{W_0 \cdot x^2}{2L}$ (παραβολή). Επιβάδον!

$M(x) = \int V dx = \frac{W_0}{2L}\frac{x^3}{3}$ κυβική παραβολή!

5-4/379



$R_A = WL$

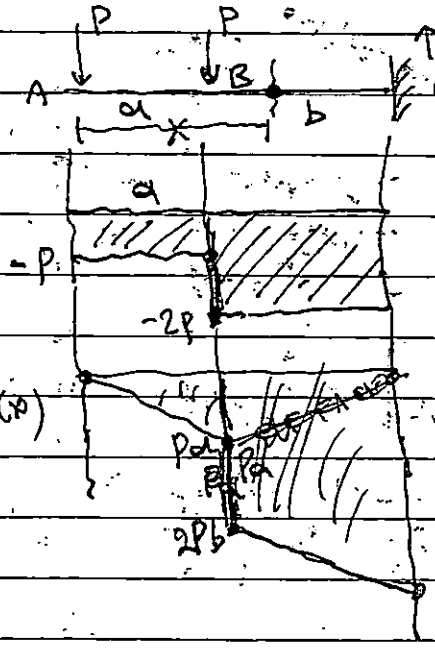
$M_A = \frac{WL}{2} \cdot \frac{L}{2} = \frac{WL^2}{4}$

$V = \int w \cdot x \, dx = \frac{wx^2}{2}$ max $\frac{L}{2}$

$V_B = \frac{WL^2}{2}$

$M = \int \frac{wx^2}{2} dx = \frac{wx^3}{6}$

5.5/379



$R_C = 2P$ $\quad a+b=L$

$PL + P(b) = M_C$

$PL + P(L-a) = M_C =$ $\quad (2PL - Pa)$

$M = -2P(x-a) + Pa$

$a \le x \le a+b$

$x = a \rightarrow M = Pa$

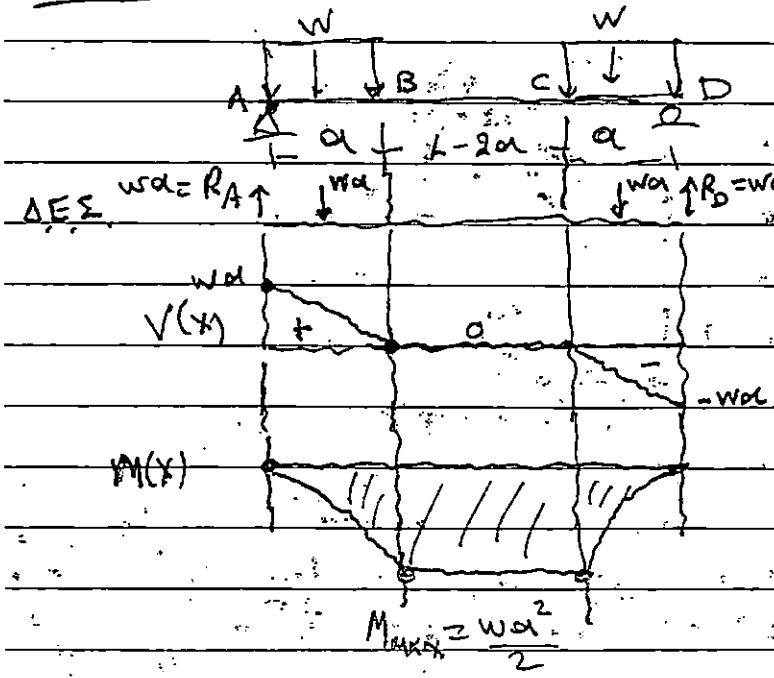
$x = a+b \rightarrow M = -2P(a+b-a) + Pa$

$= Pa - 2Pb$

$= -Pa - 2P(L-a) = Pa + 2Pa - 2PL$

$M_C = 3Pa - 2PL$

5.6/379



$\Delta E \Sigma$

Σ Totzmy: $R_A + R_D = 2wa$

$0 = \Sigma M_A \Rightarrow wa\frac{a}{2} + wa\left(a + L - 2a + \frac{a}{2}\right) = R_D L$ $\quad L - \frac{1}{2}a$

$\frac{wa^2}{2} + wa\left(L - \frac{a}{2}\right) = R_D L$

$wa\not{L} = R_D \not{L} \Rightarrow R_D = wa$

$R_A = wa$

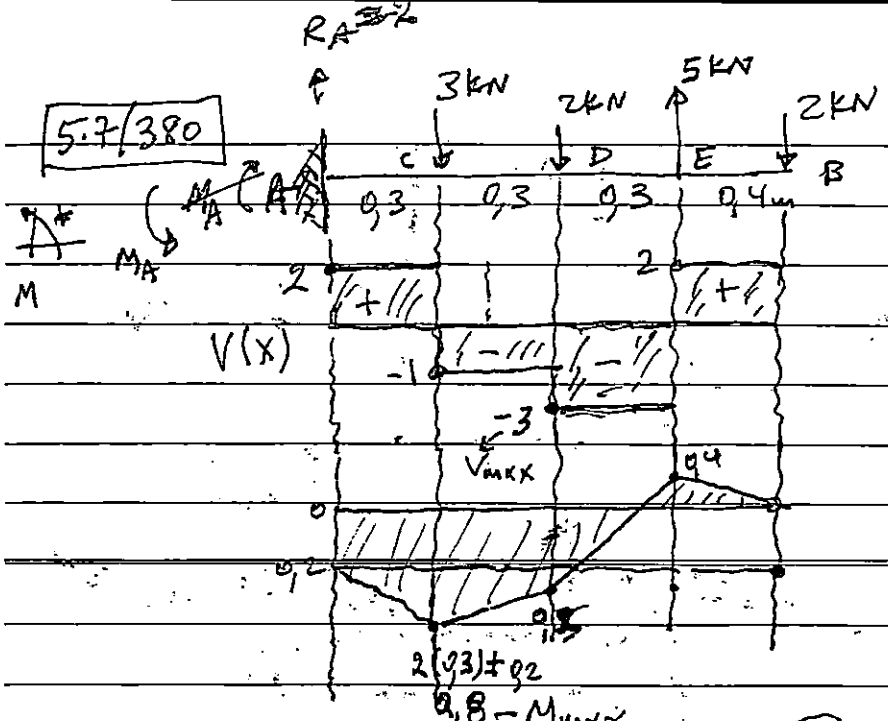
$V(x) = wa - w \cdot x$

$V_I = w(a-x)$

$M(x) = \int w(a-x)\,dx = wax - \frac{wx^2}{2}$

$M(x=a) = wa^2 - w\frac{a^2}{2} = \frac{wa^2}{2}$

5.7/380



Σ Fx = 0 ; $R_A = 3+2-5+2$

$R_A = 2$ KN

$M_A + 3(0,3) + 2(0,6) - 5(0,9) + 2(1,3) = 0$

$M_A + 0,9 + 1,2 = 4,5 + 2,6 = 0$

$M_A = -0,2$ KN.M

$M_I = 0,2 + 2(x)$

$x=0 \quad M=0,2$

$x=0,3 \quad M=0,8$

$M_{II} = 0,8 - 1(x)$

$x = \quad - M_c = 0,8$

$x = 0,8 \quad M = 0,8$

$M_{III} = 0,8 - 3(x)$

$x = \quad \rightarrow M = 0,8$

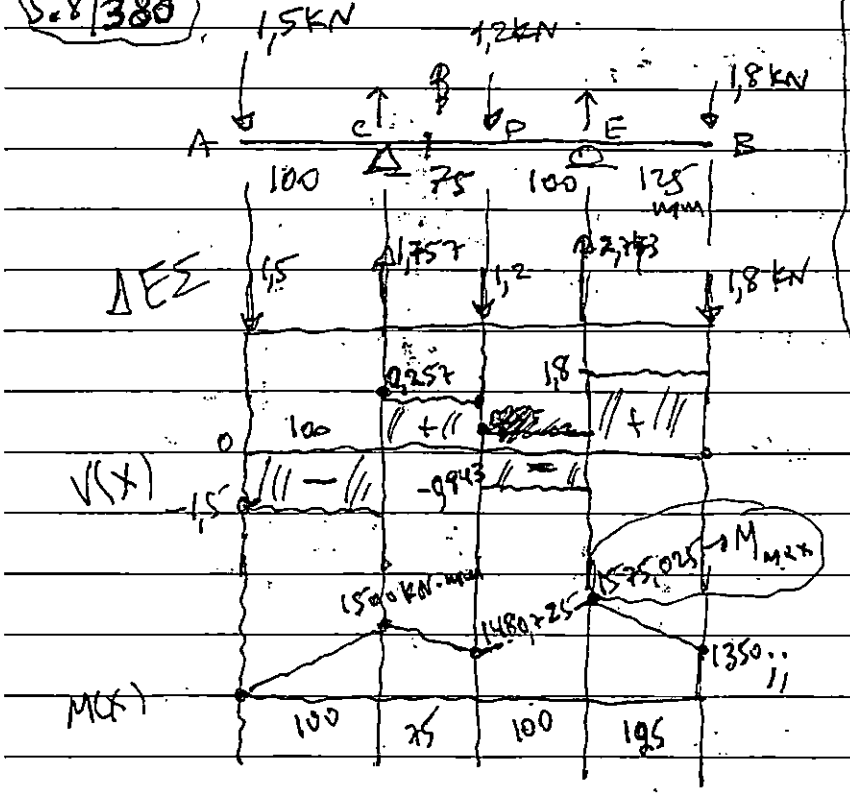
$X = 0,3 \rightarrow M = 0,8 - 0,9 = -0,4$

$M_{IV} = -0,4 + 2x$

$x=0 \rightarrow M = -0,4$

$x = 0,4 \quad M = -0,4 + 2(0,4)$

5.8/380



Σ Fx = 0 : $R_C + R_E = 1,5 + 1,2 + 1,8 = 4,5$

$\Sigma M_C = 0 \rightarrow 1,5(100) + R_E(175) = 1,2(75) +$

$R_E = \frac{90 + 540 - 150}{175} = 1,8(300)$

$R_E = 2,743$ KN ; $R_C = 1,757$ KN

$M_I = 0 + 1,5 \cdot x$ $\quad x=0 \ M=0$ $\quad x=100 \ M=1500$ KN.M

$M_{II} = 1500 - 0,257(x-100)$

$x=100 \rightarrow 1500$

$x=175 \rightarrow 1480,725$

$M_{III} = 1480,725 + 0,943(x-175)$

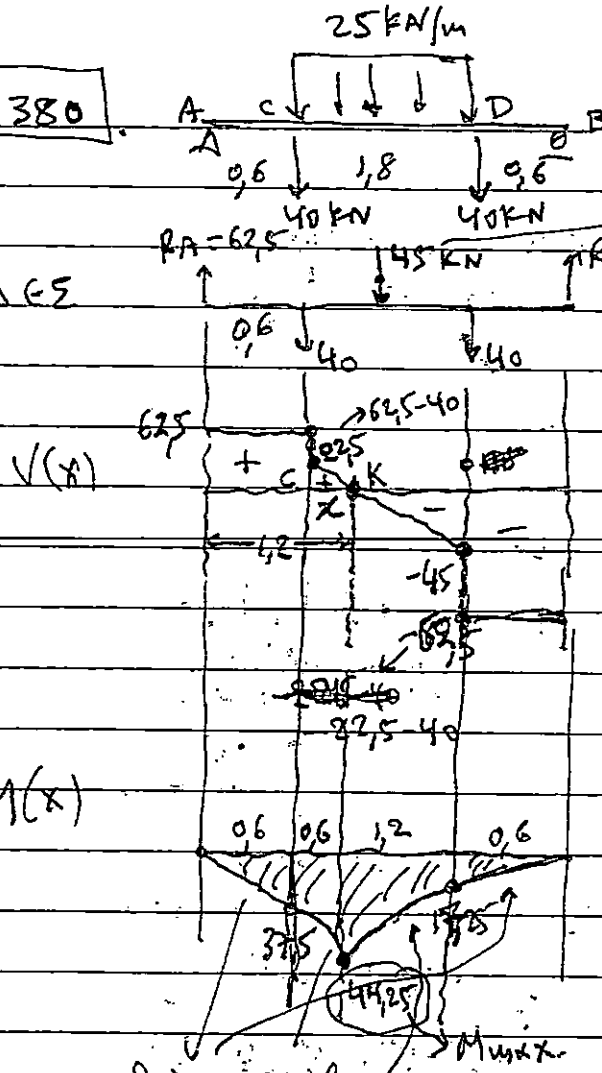
$x=175 \rightarrow M = 1480,725$

$x=275 \rightarrow M = 1575,025$

$M_{IV} = 1575,025 - 1,8(x-2,75)$

$x=500 \rightarrow M = 1575,025 - 295 \approx 1350$

5.9 | 380



Σταθμική: $R_A + R_B = 125$

$\Sigma M_A = 0^+ \rightarrow R_B(3) = 40(0,6) + 45(1,5) + 40(2,4) =$

$R_B = 62,5$ kN

$R_A = 62,5$ kN

$V_I = 62,5$

$V_{II}(x) = 62,5 - 25(x-0,6)$

$x = 0,6 \rightarrow V_{II} = 62,5$ ✓

$x = 2,4 \rightarrow V_{II} = 22,5 - 25(1,8) = -22,5$

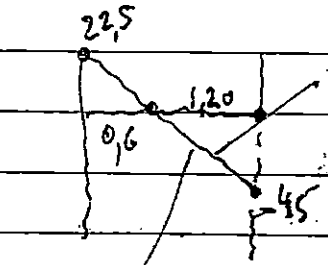
$V_{III} = -22,5 - 40 = -62,5$

$M_I = 62,5 \times 0,6 = 37,5$

Προσδιορισμός συντεταγμένων κ.

$\frac{22,5}{x} = \frac{45}{1,8-x} \Rightarrow 1,8 - x = 2x$

$x = \frac{1,8}{3} = 0,6$



$V = 22,5 - \frac{22,5}{0,6} x$

$V = A + B \cdot x$

$x = 0 \rightarrow V = 22,5 = A$

$x = 1,8 \rightarrow V = -45 = 22,5 + B(1,8)$

$B = -37,5$

$V = 22,5 - 37,5x$

$V = -\frac{45}{1,20} x = -37,5x$

$-\frac{37,5x^2}{2}$

$M = \int V dx = 22,5x - \frac{37,5x^2}{2}$

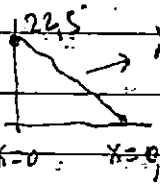
$M_{II} = 37,5 + 22,5x - \frac{37,5x^2}{2}$

$x = 0 \quad 37,5$

$x = 0,6 \quad 44,25$

$x = 1,8 \quad 37,5 + 22,5(1,8) - 60,75 = 78 - 60,75 = 17,25$

$M_{CK} = 37,5 - \frac{1}{2} 22,5 \cdot x^2$



$M_{II} = 22,5 - \frac{22,5}{0,6} \cdot x$

$M_{II} = 22,5 - 37,5x$

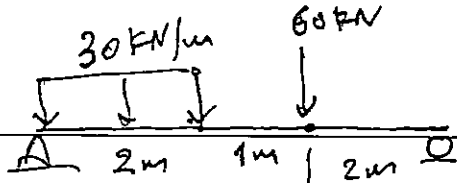
$M_{II} = \int V_{II} dx = 22,5x - \frac{37,5x^2}{2} + 37,5$

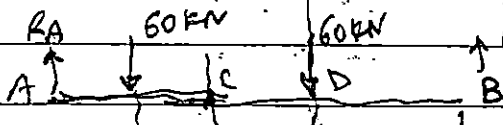
$x = 0 \rightarrow M = 37,5$ ✓

$x = 0,6 \rightarrow 37,5 + 22,5(0,6) - \frac{37,5(0,6)^2}{2} =$

44,25

5.10 | 380





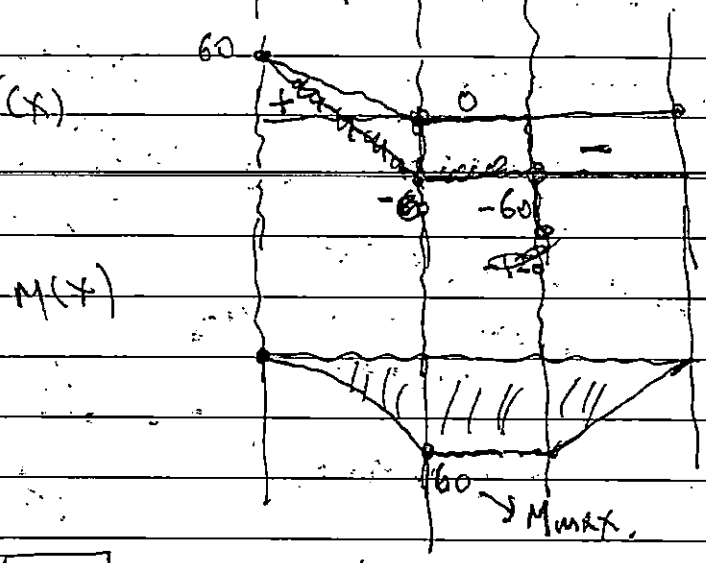
A.E.Σ.

Σ forças verticais: $R_A + R_B = 120$

$60(2) + 60(3) = R_B(5)$

$\frac{300}{5} = R_B = 60\ KN$

$R_A = 60\ KN$



$V_1 = 60 - 30x$

$x=0 \rightarrow 60$

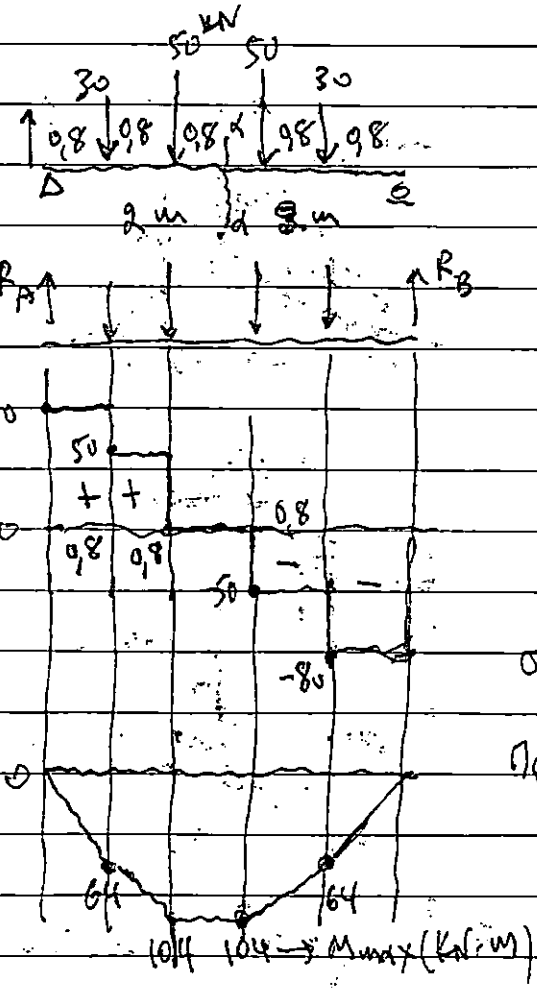
$x=2 \rightarrow -60$

$M(x) = \int_0^2 V\,dx = 60x - \frac{30}{2}x^2$

$x=0 \rightarrow M=0$

$x=2 \rightarrow M=-6$

5.18 | 381



$R_A + R_B = 160$

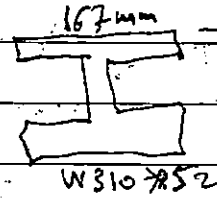
$30(0,8) + 50(1,6) + 50(2,4) + 30(3,2) = R_B(4)$

$\frac{120 + 200}{4} = R_B = 80\ KN$

$R_A = 80\ KN$

$\sigma = \frac{M_{max}}{I_{LN}} y_{max}$

Propriedades da seção I_{LN}

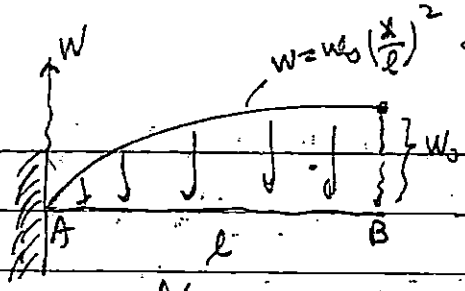


$I_{LN} = 119 \cdot 10^6\ mm^4$

$y_{max} = \frac{318}{2} = 159\ mm$

$\therefore \sigma_{max} = \frac{104 \cdot 10^6\ N.mm}{119 \cdot 10^6\ mm^4} 159\ mm = 138{,}9\ MPa$

5.50 | 393



Νά δείξετε τή Μαθημα-τική Ι.

$$V(x) = \int_0^x W(x)\,dx$$

εμβαδόν φορτίου από $0 \to x$

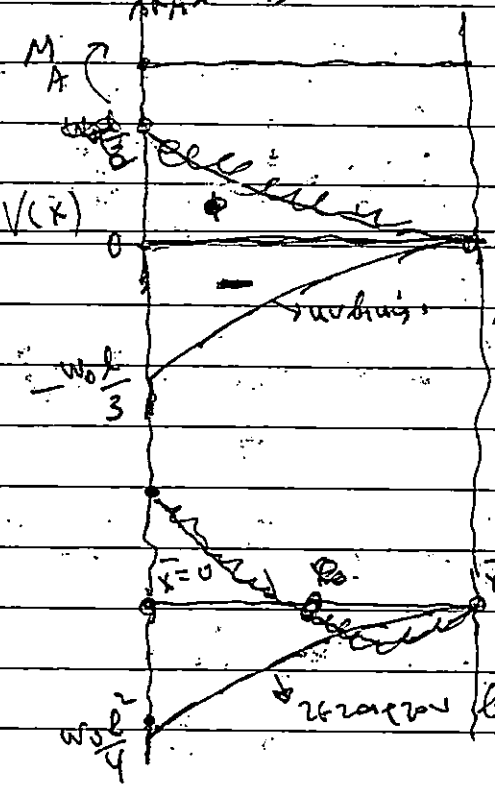
$$V(x) = \frac{W_0}{\ell^2}\,\frac{x^3}{3}$$ "παβλιάν"

$$R_A = V(x=\ell) = W_0\,\frac{\ell}{3}$$ αλλιώς φορτίο

$$V = V_A = + \frac{W_0}{\ell^2}\,\frac{x^3}{3}$$

$$V(x) = W_0\,\frac{\ell}{3} - \frac{W_0}{3}\left(\frac{x}{\ell}\right)^3$$

$$\boxed{V(x) = W_0\,\frac{\ell}{3}\left[1 - \left(\frac{x}{\ell}\right)^3\right]}$$ ok.

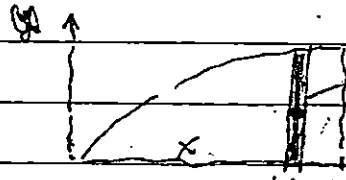


$$M - M_A = \int_0^x V(x)\,dx$$

$$= W_0\,\frac{\ell}{3}\left(\frac{x}{\ell} - \frac{\ell}{4}\,\frac{x^4}{\ell^4}\right)$$

$$W_0\,\frac{\ell^2}{3}\left[\bar{x} - \frac{1}{4}\,\bar{x}^4\right]$$

Για να υπολογίσετε τη ροπή αδράνειας M_A στην οριακή περίπτωση να προσδιορίσετε το κ.β. της επιφάνειας και φορτίου.



$$M = M_A + W_0\,\frac{\ell^2}{3}\,\bar{x}\left(1 - \frac{1}{4}\,\bar{x}^3\right)$$

τρίτου βαθμού.

$$\bar{x}_{\text{κβ}} = \frac{\int x\,dA}{\int dA} \to \frac{\int W_0\left(\frac{x}{\ell}\right)^2 dx}{\int W_0\left(\frac{x}{\ell}\right)^2 dx} =$$

$$\bar{x} = \frac{\frac{W_0}{\ell^2}\cdot\frac{\ell^4}{4}}{\frac{W_0}{\ell^2}\,\frac{\ell^3}{3}} = \frac{\ell^2/4}{\ell/3} = \frac{3}{4}\,\ell.$$

$$\therefore M_A = W_0\,\frac{\ell}{3}\cdot\frac{3}{4}\,\ell = \frac{W_0\ell^2}{4}\,\ell^2$$

Άρα $M = W_0\,\frac{\ell^2}{4} + W_0\,\frac{\ell^2}{3}\left(\frac{x}{\ell}\right)\left[1 - \frac{1}{4}\left(\frac{x}{\ell}\right)^3\right]$

Βρίσκω $\left(\frac{x}{\ell}\right)^3 = 4 \Rightarrow$

$x_0 = \ell\sqrt[3]{4} = 1{,}587\,\ell$ μιση

Γενικώς περιοχή

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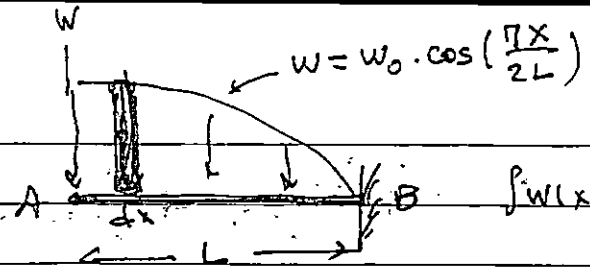


$$\int w(x)\,dx = w_0 \int_0^L \cos\left(\frac{\pi x}{2L}\right) dx$$

$$V_{max} = \frac{2L w_0}{\pi} \int_0^L \cos\left(\frac{\pi x}{2L}\right) d\left(\frac{\pi}{2L} x\right)$$

Τεχνάσμα κέντρου βάρους!

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{w_0 \int_0^L x \cos\left(\frac{\pi x}{2L}\right) dx}{\frac{2L w_0}{\pi}}$$

$$\frac{2L}{\pi}\frac{2L}{\pi} w_0 \int \left(\frac{\pi}{2L} x\right) \cos\left(\frac{\pi x}{2L}\right) d\left(\frac{\pi}{2L} x\right)$$

$$\left(\frac{2L}{\pi}\right)^2 w_0 \int_0^{\pi/2} Y \cos Y\,dY \qquad Y = \frac{\pi x}{2L}$$

$$\int Y\,d\sin(Y) = Y\sin Y - \int \sin Y\,dY$$

$$= Y\sin Y \Big|_0^{\pi/2} + \cos Y \Big|_0^{\pi/2} = \frac{\pi}{2} - 1$$

$$\bar{x} = \frac{\left(\frac{2L}{\pi}\right)^2 w_0 \left(\frac{\pi}{2} - 1\right)}{\left(\frac{2L}{\pi}\right) w_0}$$

$$= \frac{2L}{\pi}\left(\frac{\pi}{2} - 1\right) = L - \frac{2L}{\pi} = \boxed{0{,}36\,L = \bar{x}}$$

$$V_{0\lambda.} = \frac{2L w_0}{\pi}$$

$$M_B = 0{,}64L \times V_{0\lambda} = 0{,}64L \cdot \frac{2L}{\pi} w_0 = \boxed{0{,}40\,L^2 w_0}$$

0,36 0,64

Βοηθάει στην πάκτωση!

$$\left(\frac{2}{\pi}\right)^2$$

$$M(x) - M_B = \int_0^L V(x)\,dx$$

$$\int_0^x w(x)\,dx = w_0 \frac{2L}{\pi} \int_0^x \cos\left(\frac{\pi x}{2L}\right) d\left(\frac{\pi x}{2L}\right)$$

$$= \left(\frac{2L}{\pi}\right)^2 w_0 \int_0^x \sin\frac{\pi x}{2L}\, d\left(\frac{\pi}{2L} x\right)$$

$$\frac{2L w_0}{\pi} \left[\sin\left(\frac{\pi x}{2L}\right)\right]_0^x$$

$$= \left(\frac{2L}{\pi}\right)^2 w_0 \left[-\cos\left(\frac{\pi x}{2L}\right)\right]_0^x \qquad V(x) = \frac{2L}{\pi} w_0 \sin\left(\frac{\pi x}{2L}\right)$$

$$= \left(\frac{2L}{\pi}\right)^2 w_0 \left(1 - \cos\left(\frac{\pi x}{2L}\right)\right)$$

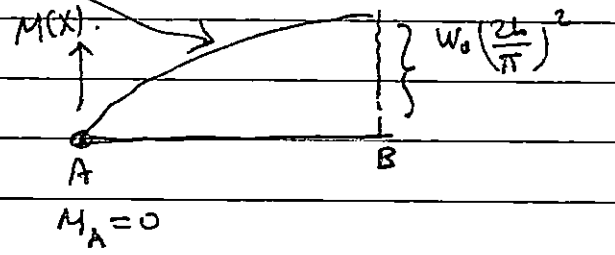
$$\therefore M(x) = \left(\frac{2}{\pi}\right)^2 L^2 w_0 + \left(\frac{2L}{\pi}\right)^2 w_0 \left(1 - \cos\frac{\pi x}{2L}\right)$$

~~Κάνω από το σημείο Β.~~

$$M(x) = \left(\frac{2L}{\pi}\right)^2 w_0 \left[1 - \cos\left(\frac{\pi x}{2L}\right)\right]$$

για $x = 0$ (δεξιά πάκτωση) $\quad M_B = w_0 \left(\frac{2L}{\pi}\right)^2$

$x = L$ (ελεύθερο άκρο) $\quad M = M_B$



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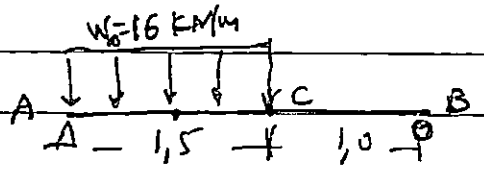
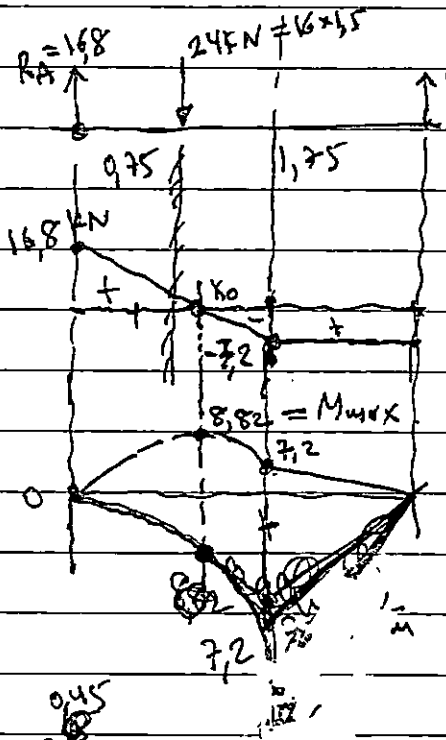


ΔΕΣ



6Ε] 899.

S 15 8 × 18,6

$I = 9,16 \times 10^6\ mm^4$

$d = 152\ mm$

$R_A + R_B = 24$

$24\left(\frac{1,5}{2}\right) = R_B \cdot (2,5) \Rightarrow$

$R_B = 7,2\ kN$

$\rightarrow R_A = 16,8\ kN.$

$V(x) - V_A = -\int w_0 \cdot dx = 16 \cdot x$

$V(x) = 16,8 - 16x$

$V(x) = 0 \rightarrow x_0 = \frac{16,8}{16} = 1,05\ m$

$V_C = 16,8 - 16(1,5) = -7,2$

$M - M_A = \int_0^x V(x)\,dx$

$M - M_A = 16,8x - 16\frac{x^2}{2} = M(x)$

κάθετη. (παραβολή)

$M_C = 16,8(1,5) - 8(1,5)^2 = 7,2\ kN.m$

$M_0 = 16,8(1,05) - 8(1,05)^2 = 8,82\ kN.m$

$M_C - M_0 = \int_{0,45}^{1,5} V(x)\,dx$

$\left(16,8x - 8x^2\right)_{0,45}^{1,5}$

$16,8(0,45) - 8(0,45)^2 = 5,94$

$\therefore M_C = 8,82 + 5,94 = 14,76\ kN.m$

$\sigma = \frac{M_{max}}{I} \cdot y_{max} = \frac{8,82 \cdot 10^6\ N.mm}{9,16\ 10^6} \cdot \frac{152}{2}\ mm = 73,2\ MPa$

($y_{max} = d/2$)