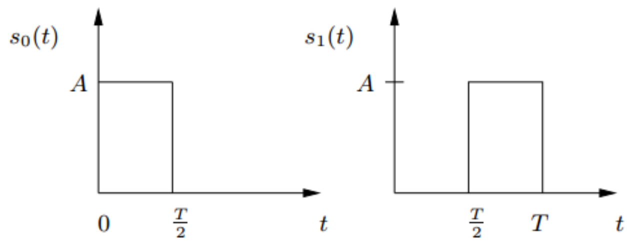
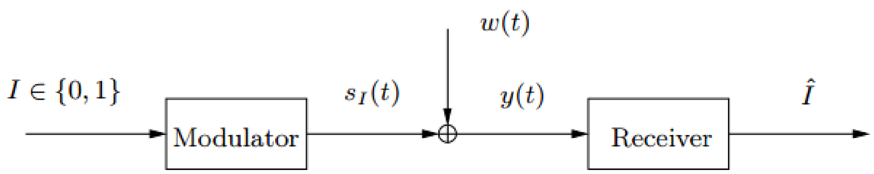
3. The information variable $I \in \{0, 1\}$ is mapped onto the signals $s_0(t)$ and $s_1(t)$.



The probabilities of the outcomes of I are given as

$$\begin{aligned} \Pr(I = 0) = \Pr(s_0(t) \text{ is transmitted}) &= p \\ \Pr(I = 1) = \Pr(s_1(t) \text{ is transmitted}) &= 1 - p \end{aligned}$$

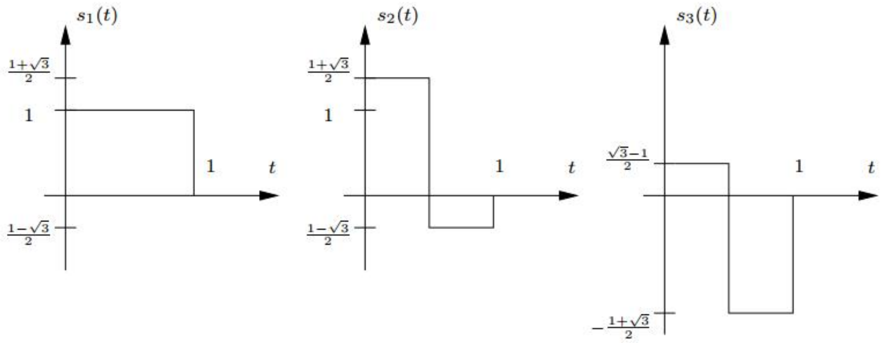
The signal is transmitted over a channel with additive white Gaussian noise, $W(t)$, with power spectral density $N_0/2$.



(a) Find the receiver (demodulator and detector) that minimizes $\Pr(\hat{I} \neq I)$.

(b) Find the range of different p's for which $y(t) = s_1(t) \Rightarrow \hat{I} = 0$.

3. Consider the six signal alternatives $s_1(t), \ldots, s_6(t)$, where $s_1(t), s_2(t), s_3(t)$ are shown below,



and with

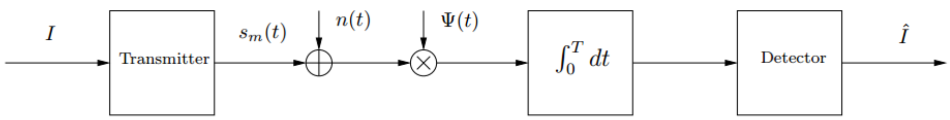
$$s_4(t) = -s_1(t), \quad s_5(t) = -s_2(t), \quad s_6(t) = -s_3(t)$$

These signals are employed in transmitting equally probable symbols over an AWGN channel, with noise spectral density $N_0/2$ and using an optimal (minimum symbol error probability) receiver.

With $P_e = \Pr(\text{symbol error})$, show that

$$Q\left(\frac{1}{\sqrt{2N_0}}\right) < P_e < 2\,Q\left(\frac{1}{\sqrt{2N_0}}\right)$$

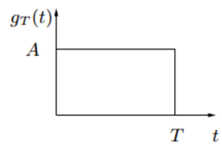
3. Consider the baseband PAM system below, using a correlation-type demodulator. $n(t)$ is AWGN with spectral density $N_0/2$ and $\Psi(t)$ is the unit energy basis function. The receiver uses ML detection.



The transmitter maps each source symbol I onto one of the equiprobable waveforms $s_m(t)$, $m = 1 \ldots 4$.

$$
\begin{aligned}
s_1(t) &= \frac{3}{2} g_T(t) \\
s_2(t) &= \frac{1}{2} g_T(t) \\
s_3(t) &= -s_2(t) \\
s_4(t) &= -s_1(t)
\end{aligned}
$$

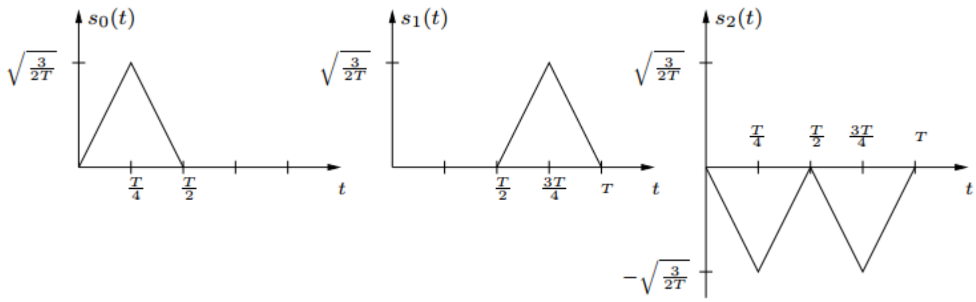
where $g_T(t)$ is a rectangular pulse of amplitude A.



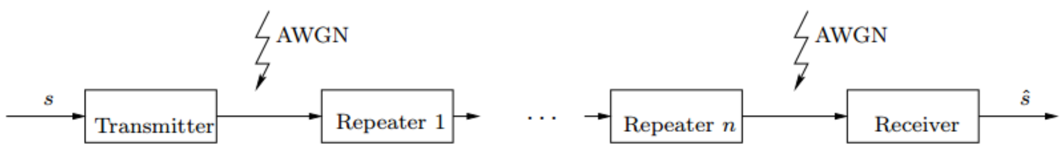
However, due to manufacturing problems, the amplitude of the used basis function is corrupted by a factor $b > 1$. I.e. the used basis function is $\bar{\Psi}(t) = b\Psi(t)$. Note that the detector is unchanged.

Compute the symbol-error probability ($\Pr(\hat{I} \neq I)$).

3. Consider the waveforms $s_0(t)$, $s_1(t)$ and $s_2(t)$.



They are used to transmit symbols in a communication system with n repeaters. Each repeater estimates the transmitted symbol from the previous transmitter and retransmits it. Assume ML detection in all receivers.



Assume that $\Pr\{s_0(t) \text{ transmitted}\} = \Pr\{s_1(t) \text{ transmitted}\} = \frac{1}{2}\Pr\{s_2(t) \text{ transmitted}\} = \frac{1}{4}$ and that each link is disturbed by additive white Gaussian noise with spectral density $N_0/2$. Derive an upper bound to the total probability of error, $\Pr(\hat{s} \neq s)$.

3. Consider binary antipodal signaling with equally probable waveforms

$$s_1(t) = 0, \quad 0 \le t \le T$$
$$s_2(t) = \sqrt{\frac{E}{T}}, \quad 0 \le t \le T$$

in AWGN with spectral density $N_0/2$.

The optimal receiver can be implemented using a matched filter with impulse reponse

$$h_{\text{opt}}(t) = \sqrt{\frac{1}{T}}, \quad 0 \le t \le T$$

sampled at $t = T$. However in this problem we consider using the (suboptimal) filter

$$h(t) = e^{-t/T}, \quad t \ge 0$$

($h(t) = 0$ for $t < 0$) instead of the macthed filter. More precicely, letting y_T denote the value of the output of this filter sampled at $t = T$, when fed by the received signal in AWGN, the decision is

$$y_T < b \implies \text{choose } s_1$$
$$y_T \ge b \implies \text{choose } s_2$$

where $b > 0$ is a decision threshold.

(a) Determine the resulting error probability P_e, as a function of b, E, T and N_0. (3p)

(b) Which value for the threshold b minimizes P_e?

3. Let three orthonormal waveforms be defined as

$$\psi_1(t) = \begin{cases} \sqrt{\frac{3}{T}}, & 0 \le t < \frac{T}{3} \\ 0, & \text{otherwise} \end{cases} \quad \psi_2(t) = \begin{cases} \sqrt{\frac{3}{T}}, & \frac{T}{3} \le t < \frac{2T}{3} \\ 0, & \text{otherwise} \end{cases} \quad \psi_3(t) = \begin{cases} \sqrt{\frac{3}{T}}, & \frac{2T}{3} \le t < T \\ 0, & \text{otherwise} \end{cases}$$

and consider the three signal waveforms

$$s_1(t) = A\left(\psi_1(t) + \frac{3}{4}\,\psi_2(t) + \frac{\sqrt{3}}{4}\,\psi_3(t)\right)$$
$$s_2(t) = A\left(-\psi_1(t) + \frac{3}{4}\,\psi_2(t) + \frac{\sqrt{3}}{4}\,\psi_3(t)\right)$$
$$s_3(t) = A\left(-\frac{3}{4}\,\psi_2(t) - \frac{\sqrt{3}}{4}\,\psi_3(t)\right)$$

Assume that these signals are used to transmit equally likely symbol alternatives over an AWGN channel with noise spectral density $N_0/2$.

(a) Show that optimal decisions (minimum probability of symbol error) can be obtained via the outputs of *two* correlators (or sampled matched filters) and specify the waveforms used in these correlators (or the impulse responses of the filters). (2p)

(b) Assume that P_e is the resulting probability of symbol error when optimal demodulation and detection is employed. Show that

$$Q\left(\sqrt{\frac{2A^2}{N_0}}\right) < P_e < 2Q\left(\sqrt{\frac{2A^2}{N_0}}\right)$$

(1p)

(c) Use the bounds in (b) to upper-bound the symbol-rate (symbols/s) that can be transmitted, under the constraint that $P_e < 10^{-4}$, and that the average transmitted power is less than or equal to P. Express the bound in terms of P and N_0 (2p)