

Principles of Wireless Sensor Networks

<https://kth.instructure.com/courses/2912/>

Lecture 10

Positioning and Localization

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Course content

- Part 1

- ▶ Lec 1: Introduction to WSNs

- Part 2

- ▶ Lec 2: Wireless Channel
- ▶ Lec 3: Physical Layer
- ▶ Lec 4: Medium Access Control Layer
- ▶ Lec 5: Routing

- Part 3

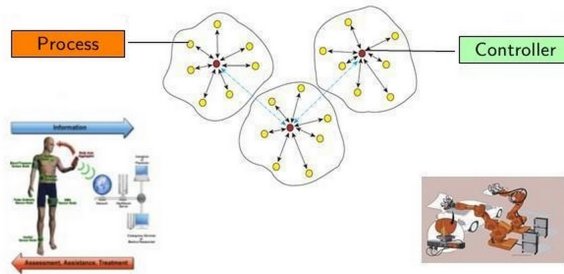
- ▶ Lec 6: Distributed Detection
- ▶ Lec 7: Seminars from Industry
- ▶ Lec 8: Static Distributed Estimation
- ▶ Lec 9: Dynamic Distributed Estimation
- ▶ Lec 10: Positioning and Localization
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- Part 4

- ▶ Lec 12: Wireless Sensor Network Control Systems 1
- ▶ Lec 13: Wireless Sensor Network Control Systems 2

Previous lecture

Application
Presentation
Session
Transport
Routing
MAC
Phy



How to estimate phenomena from noisy measurements?

Today's learning goals

- Which measurements are used for estimating the position of a node?
- How to estimate the position of a node?
- What is the effect of measurement errors?

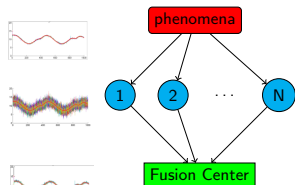
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- Estimation of the position

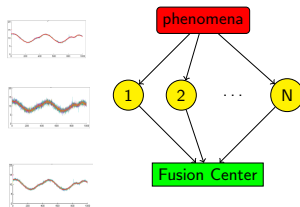
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 - ▶ Time of arrival
 - ▶ Time difference of arrival
 - ▶ Received signal strength
 - ▶ Angle of Arrival
- Estimation of the position
 - ▶ Angle of arrival + velocity
 - ▶ Triangulation
 - ▶ Trilateration
 - ▶ Iterative and collaborative multilateration

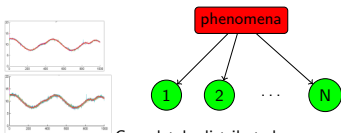
Estimation



Centralized Estimation:
no intelligence on sensors



Distributed Estimation:
some intelligence on sensors



Completely distributed
(or decentralized) Estimation



Positioning and localization

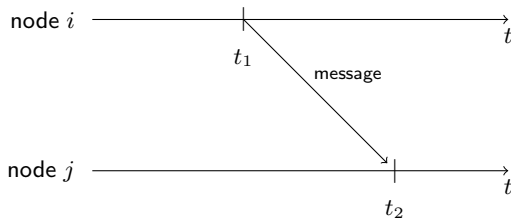
Localization is defined as a technique to estimate the positions of nodes

- It can be categorized into
 - ▶ Centralized, where a central node estimates the position of the nodes
 - ▶ Distributed, where many nodes help each other to find their own positions

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Time of arrival



- Assuming that the nodes are synchronized the distance measurement is

$$d_{ij} \simeq (t_2 - t_1) \cdot v$$

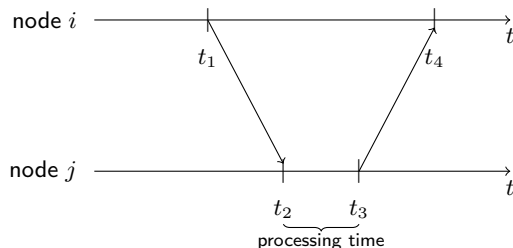
where v is the propagation speed of the message

- Uncertainty occurs when $t_2 \approx t_1$
- Problems
 - ▶ Packet losses
 - ▶ MAC delays
 - ▶ CPU delay

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Time difference of arrival



- The distance measurement is

$$d_{ij} \simeq \frac{(t_2 - t_1) + (t_4 - t_3)}{2} \cdot v$$

where v is the propagation speed of the message

- Problems
 - ▶ Packet losses
 - ▶ MAC delays
 - ▶ CPU delay

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Received signal strength (RSS)



Ideal propagation

$$P_r = P_t G_t G_r \overline{PL} \frac{\lambda^2}{(4\pi d_{ij})^2} \Rightarrow d_{ij} \simeq \frac{\lambda}{4\pi} \sqrt{\frac{P_t G_t G_r \overline{PL}}{P_r}}$$

Problems: multi-path, noise, etc, affect the propagation characteristics
 \Rightarrow Channel estimation needed

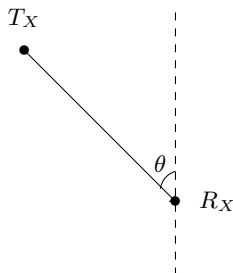
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Angle of arrival

Example: Sound propagation by multiple microphones, amplitude and phase

- Odometry sensor



Problems: Errors in measurements of θ (e.g., due to magnetic fields of earth), estimation accuracy decreases with the distance.

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Estimation of the position

We now examine the specific techniques that are used to estimate the sensors' position based on the measurements that were previously presented

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Angle of arrival + velocity


$$\tilde{\theta}(k) = \theta(k) + n_{\theta}(k)$$

- $\tilde{\theta}(k)$ the angle measurement at discrete time k given by, e.g., odometry sensor
- $\theta(k)$ the real angle value
- $n_{\theta}(k)$ the additive noise

$$\tilde{v}(k) = v(k) + n_v(k)$$

- $\tilde{v}(k)$ the velocity measurement at discrete time k given by, e.g., accelerometer
- $v(k)$ the real velocity value
- $n_v(k)$ the additive noise

Angle of arrival + velocity

Node moving
with velocity v A diagram showing a black dot representing a node, followed by a horizontal arrow pointing to the right. Below the arrow is the variable 'v'.

Let $X_r(k) = \begin{bmatrix} x(k) \\ y(k) \end{bmatrix}$ be the true position of the node that we want to estimate
Then

$$\begin{aligned}\hat{x}(k+1) &= \hat{x}(k) + \tilde{v}(k) \cdot T \cdot \cos \tilde{\theta}(k) \\ \hat{y}(k+1) &= \hat{y}(k) + \tilde{v}(k) \cdot T \cdot \sin \tilde{\theta}(k)\end{aligned}$$

where T is the sampling time

A problem is that the resulting estimator is **biased**

$$\begin{aligned}\mathbb{E}\{\hat{x}(k+1)\} &= \mathbb{E}\{\hat{x}(k)\} + \mathbb{E}\{\tilde{v}(k)\} \cdot T \cdot \mathbb{E}\{\cos \tilde{\theta}(k)\} = \\ &= \mathbb{E}\{\hat{x}(k)\} + v(k) \cdot T \cdot \cos \theta(k) \cdot e^{-\frac{\sigma_{\theta}^2}{2}} \neq x(k+1)\end{aligned}$$

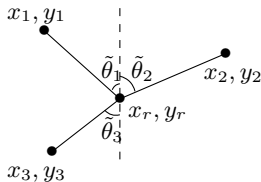
We need to estimate the bias as well.

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Triangulation

- Anchors: nodes with fixed position used for determining the unknown position of a node



Let $X_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$ be the true position of the node that we want to estimate

The angle measurements are

$$\tilde{\theta}_i = \theta_i(X_r) + n_i \quad i = 1, 2, 3$$

where $\theta_i(X_r) = \arctan \frac{x_r - x_i}{y_r - y_i}$

Triangulation

Setting

- $\underline{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ as the vector of noises with $\mathbb{E} \{n \cdot n^T\} = R = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$
- $\theta(X_r) = \begin{bmatrix} \theta_1(X_r) \\ \theta_2(X_r) \\ \theta_3(X_r) \end{bmatrix}$ as the vector of true node position and
- $Y = \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{bmatrix}$ as the vector of angle measurements

we can estimate X_r from

$$Y = \theta(X_r) + \underline{n}$$

where $\theta(X_r)$ is a non-linear function

Triangulation

Error covariance (cf. slide 19 in Lecture 8)

$$C\left(\hat{X}_r\right)=\left[\theta\left(\hat{X}_r\right)-Y\right]^T \cdot R^{-1} \cdot\left[\theta\left(\hat{X}_r\right)-Y\right]=\sum_{i=1}^3 \frac{\left(\theta_i\left(\hat{X}_r\right)-\tilde{\theta}_i\right)^2}{\sigma_i^2}$$

Goal: Choose \hat{X}_r that keeps oscillations of $\theta\left(\hat{X}_r\right)$ around Y

Therefore

$$\min _{\hat{X}_r} C\left(\hat{X}_r\right) \equiv\left\{\begin{array}{l} \frac{d C\left(\hat{X}_r\right)}{d x_r}=0 \\ \frac{d C\left(\hat{X}_r\right)}{d y_r}=0 \end{array}\right\}$$

Triangulation

We can use iterative methods for solving the above system of non-linear equations, e.g.,

Newton-Gauss method

Consider the non-linear equation

$$g(x) = 0 \quad g : \mathbb{R}^N \rightarrow \mathbb{R}^N \quad x \in \mathbb{R}^N$$

Iterative method $x_{t+1} = x_t - \delta_t \cdot g(x_t)$

where $\delta_t = \nabla^{-1} g(x_t)$

$$\lim_{t \rightarrow \infty} x_t \rightarrow x^* : g(x^*) = 0$$

∇ is the matrix of all first-order partial derivatives (Jacobian)

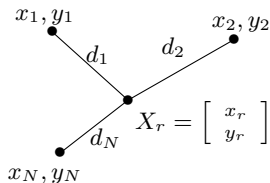
In this specific case, we use the iterative method to solve

$$g(x) = \nabla_x C(\hat{X}_r) = 0$$

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Trilateration



Let $X_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$ be the true position of the node that we want to estimate

The distance measurements are

$$\tilde{d}_i = d_i + n_i \quad i = 1, \dots, N$$

From the trigonometry

$$\begin{aligned} (x_1 - x_r)^2 + (y_1 - y_r)^2 &= \tilde{d}_1^2 \\ &\vdots \\ (x_N - x_r)^2 + (y_N - y_r)^2 &= \tilde{d}_N^2 \end{aligned}$$

Trilateration

After subtracting these N equations we arrive at the following system of equations

$$A \cdot X_r = Y$$

where $A \in \mathbb{R}^{(N-1) \times 2}$, $X_r \in \mathbb{R}^2$, $Y \in \mathbb{R}^{(N-1) \times 1}$

$$A = 2 \cdot \begin{bmatrix} (x_N - x_1) & (y_N - y_1) \\ \vdots & \vdots \\ (x_N - x_{N-1}) & (y_N - y_{N-1}) \end{bmatrix}$$

$$Y = \begin{bmatrix} \tilde{d}_1^2 - \tilde{d}_N^2 - x_1^2 - y_1^2 + x_N^2 + y_N^2 \\ \vdots \\ \tilde{d}_{N-1}^2 - \tilde{d}_N^2 - x_{N-1}^2 - y_{N-1}^2 + x_N^2 + y_N^2 \end{bmatrix}$$

Trilateration

From the previous lecture we considered a measurements model $Y = HX + n$, but we now have

$$Y = AX_r$$

A is not a square matrix and there is no explicit noise (all is included in Y).
 X_r is constant

We can apply the Linear Minimum Mean Squared Estimator (LMMSE) with

$$\hat{X}_r = L \cdot Y$$

where

$$L = \left(A^T A \right)^{-1} A^T$$

Trilateration

We can redefine \hat{X}_r as a **least square** solution.

This is equivalent to define the following cost function

$$C(X_r) = (AX_r - Y)^T (AX_r - Y)$$

and search for X_r that minimizes $C(X_r)$

$$\frac{dC(X_r)}{dX_r} = 2A^T (A\hat{X}_r - Y) = 0 \Rightarrow A^T A\hat{X}_r = A^T Y \Rightarrow \hat{X}_r = (A^T A)^{-1} A^T Y$$

Trilateration

Note that \hat{X}_r is a random variable due to Y being noisy

Taking the expected values,

$$\mathbb{E}\{\hat{X}_r\} = (A^T A)^{-1} A^T \mathbb{E}\{Y\}$$

$$\mathbb{E}\{Y\} = \begin{bmatrix} \mathbb{E}\{\tilde{d}_1^2\} - \mathbb{E}\{\tilde{d}_N^2\} - x_1^2 - y_1^2 + x_N^2 + y_N^2 \\ \vdots \\ \mathbb{E}\{\tilde{d}_{N-1}^2\} - \mathbb{E}\{\tilde{d}_N^2\} - x_{N-1}^2 - y_{N-1}^2 + x_N^2 + y_N^2 \end{bmatrix}$$

$$\tilde{d}_i = d_i + n_i \Rightarrow \tilde{d}_i^2 = d_i^2 + n_i^2 + 2d_i n_i \Rightarrow E\{\tilde{d}_i^2\} = d_i^2 + E\{n_i^2\} = d_i^2 + \sigma_i^2$$

where $\sigma_i^2 = \sigma_0^2 e^{k_\sigma d_i}$

The result is that \hat{X}_r is a biased estimator since

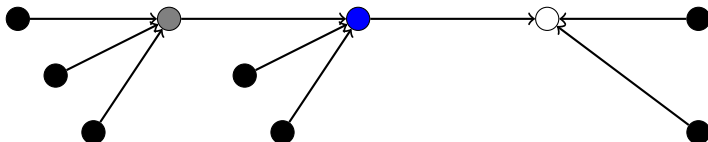
$$\mathbb{E}\{\hat{X}_r\} = (A^T A)^{-1} A^T \mathbb{E}\{Y\} \neq X_r$$

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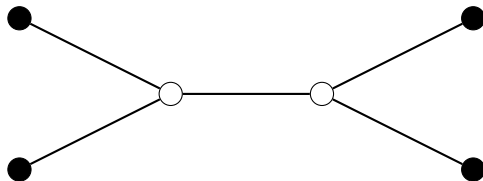
Iterative and collaborative multilateration

- In lateration techniques, at least three anchor nodes are required for estimating position
- Iterative and collaborative multilateration are extension of lateration that do not require three neighbouring anchors
- In iterative multilateration
 - ▶ Any node can become anchor after estimating its position and send anchor messages on the network
 - ▶ In this way, all nodes can estimate their positions after several iterations
- For example, in the figure below
 - ▶ Gray node estimates its position by three black anchors
 - ▶ Blue node estimates by gray and two black anchor nodes
 - ▶ White node is localized by blue and two black anchor nodes



Iterative and collaborative multilateration

- It is possible that nodes cannot have three anchors even after several iterations
- In that case collaborative multilateration is used in which
 - ▶ A graph of participating nodes is constructed
 - ▶ Participating nodes are the ones that are either anchors or have at least three participating neighbours
 - ▶ This gives set of over-constrained quadratic equations relating distance among nodes and their neighbours
 - ▶ These equation are solved to estimate positions



Summary

- We have studied the basic of localization for sensor networks
- Localizing the nodes consists in applying estimation techniques

Next lecture

- Application of estimation and detection to synchronization