Principles of Wireless Sensor Networks

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Lecture 13

Wireless Sensor Network Control Systems 2

Carlo Fischione
Associate Professor of Sensor Networks
e-mail:carlofi@kth.se
http://www.ee.kth.se/∼carlofi/

KTH Royal Institute of Technology
Stockholm, Sweden

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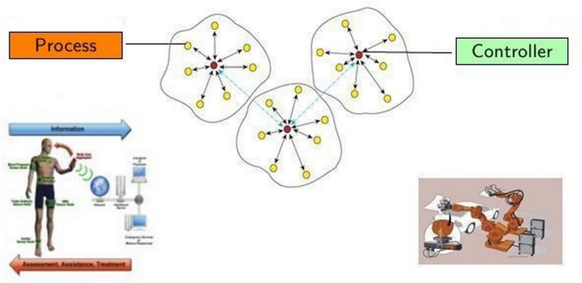
Course content

- Part 1
  - Lec 1: Introduction to WSNs
- Part 2
  - Lec 2: Wireless Channel
  - Lec 3: Physical Layer
  - Lec 4: Medium Access Control Layer
  - Lec 5: Routing
- Part 3
  - Lec 6: Distributed Detection
  - Lec 7: External Seminars
  - Lec 8: Static Distributed Estimation
  - Lec 9: Dynamic Distributed Estimation
  - Lec 10: Positioning and Localization
  - Lec 11: Time Synchronization
- Part 4
  - Lec 12: Wireless Sensor Network Control Systems 1
  - Lec 13: Wireless Sensor Network Control Systems 2
  - Lec 14: October 11, Summary and Project Presentations

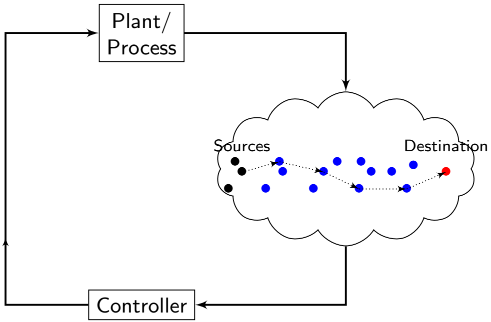
Previous lecture

Application
Presentation
Session
Transport
Routing
MAC
Phy



How to model mathematically a closed loop control system?

Today's learning goals

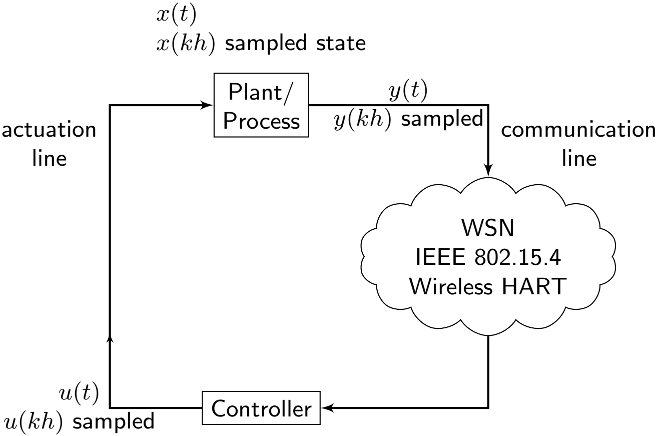


- How stability is affected by delays introduced by the WSN?
- How stability is affected by packet losses introduced by the WSNs?
- How to design WSNCS?

Outline

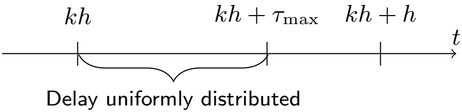
- WSNCSs with random network delay
- WSNCSs with asynchronous events
- WSNCSs with packet losses
- System Identification in WSNCSs
- Design of WSNCSs

WSNCSs



$x(t)$
$x(kh)$ sampled state
Plant/ Process
$y(t)$
$y(kh)$ sampled
communication line
actuation line
WSN
IEEE 802.15.4
Wireless HART
$u(t)$
$u(kh)$ sampled
Controller

WSNCSs with random network delay



- We now assume that the delay is not constant, but a random variable of maximum value $\tau_{\max}$
- Using $\tau_{\max}$ as in the previous case, would give conservative results
- We could exploit the randomness to get better results

When is the system stable despite random delay?

WSNCS with random network delay

Assume for simplicity $h = 1$

$$x(k+1) = \phi x(k) + \Gamma u(k)$$

$$y(k) = Cx(k) + Du(k)$$

We rewrite the system of equations in terms of z-transform,

$$zX(z) - X(0) = \phi X(z) + \Gamma U(z)$$

$$Y(z) = CX(z) + DU(z)$$

where $X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$, $U(z) = \sum_{k=0}^{\infty} u(k)z^{-k}$ and $Y(z) = \sum_{k=0}^{\infty} y(k)z^{-k}$

Note: the z-transform converts discrete-time signals into a frequency domain representation (similarly to Laplace transformation for continuous-time signals).

WSNCSs with random network delay

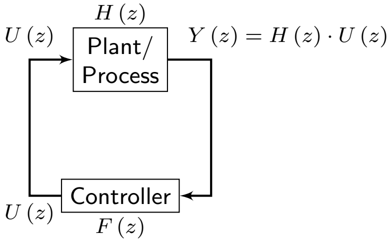
Combining the equations and solving for Y,

$$
\begin{aligned}
Y(z) &= C(zI-\phi)^{-1}zX(0) + \left[C(zI-\phi)^{-1}\Gamma + D\right]U(z) \\
&= C(zI-\phi)^{-1}zX(0) + H(z)U(z)
\end{aligned}
$$

where $H(z)$ is defined as the **pulse transfer function** of the system

$$
H(z) \triangleq \left[C(zI-\phi)^{-1}\Gamma + D\right]
$$

WSNCSs with random network delay



The control law is given as

$$u(k) = \sum_{n=0}^{k} f(n)\, y(k-n) \xrightarrow{Z\{\}} U(z) = F(z) \cdot Y(z)$$

$F(z)$ can be properly designed in order to affect the controller's decision

WSNCSs with random network delay

Theorem

Consider the WSNCS with a random uniform distributed delay with $\tau_{\max} \leq h$ *and* $U(z) = F(z) \cdot Y(z)$*. The closed loop system is stable if*

$$\left| \frac{F(z)\,H(z)}{1 + F(z)\,H(z)} \right| \leq \frac{1}{\tau_{\max} \cdot |z-1|} \quad \forall \omega$$

where $z = e^{i\omega}$

- Sufficient but not necessary condition for stability
- $F(z)$ and/or $\tau_{\max}$ can be tuned to achieve closed loop stability

Outline

- WSNCSs with random network delay
- WSNCSs with asynchronous events
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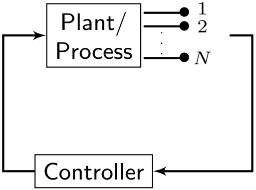
WSNCS with asynchronous events

Consider a WSNCS where the same state, $x\,(k)$, is observed at different time instants with different measurements



The system is described by the set of difference equations

$$x\,(k+1) = f_i\,(x\,(k)) \quad i = 1, ..., N$$

where $1, \ldots, N$ the set of discrete states with the respective associated rates $r_1, r_2, ..., r_N$. These rates are the fraction of time that each discrete state occurs, that is

$$r_i = \frac{1}{t_i} \quad i = 1, ..., N$$

WSNCSs with asynchronous events

The stability condition of such model is given by the following theorem

Theorem

Given a WSNCS as defined above, if there exists a function $V(x) : \mathbb{R}^n \to \mathbb{R}_+$ such that

$$V(x)|_{x=0} = 0, \quad V(x)|_{x\neq 0} > 0, \quad \dot{V}(x) < 0,$$

and scalars $\alpha_1, \alpha_2, ..., \alpha_N$ corresponding to each rate such that

$$\alpha_1^{r1} \cdot \alpha_2^{r2} \cdot \cdots \cdot \alpha_N^{rn} > \alpha > 1$$

and

$$V(x(k+1)) - V(x(k)) \leq ({\alpha_i}^{-2} - 1)V(x(k)) \quad i = 1, \dots, N$$

then the WSNCS is asymptotically stable.

$V(x)$ is the Lyapunov function of x.
α defines the decay rate of x

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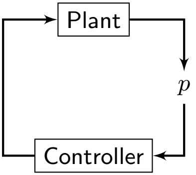
Packet losses



- The probability of a message loss is defined by p
- In the presence of message losses, we let the control input $u(t)$ be defined as $u(t) = -L\overline{x}(kh)$ for $t \in [kh, kh+h)$, where

$$\overline{x}(kh) = \begin{cases} x(kh-h) & \text{the message at time } k \text{ was lost} \\ x(kh) & \text{otherwise} \end{cases}$$

WSNCSs with packet losses

The characteristic equations of the closed loop system are

$$x\left(\left(k+1\right)h\right)=\phi x\left(kh\right)+\Gamma u\left(kh\right)$$

$$u\left(kh\right)=-L\bar{x}\left(kh\right)$$

where recall that

$$\bar{x}\left(kh\right)=\begin{cases} x\left(kh\right) & \text{if no packet losses} \\ \bar{x}\left(\left(k-1\right)h\right) & \text{otherwise} \end{cases}$$

When is the system stable?

WSNCSs with packet losses

Theorem

Suppose that the closed loop system is stable in the case of no packet losses. Then

1. *If the open loop system is stable (i.e., $\rho(\phi)$ is stable) then the closed loop system (with packet losses) is stable for every p*
2. *If ϕ is unstable, then the closed loop system (with packet losses) is stable when*

$$\frac{1}{1-\frac{\gamma_1}{\gamma_2}} < 1-p$$

where $\gamma_1 = \log \lambda^2_{\max}(\phi - \Gamma L)$, $\gamma_2 = \log \lambda^2_{\max}(\phi)$ and $\lambda_{\max}$ denoting the maximum eigenvalue

The following control and network parameters can be jointly designed to ensure stability:

- The packet loss probability p, that depends on PHY, MAC, routing
- The controller L

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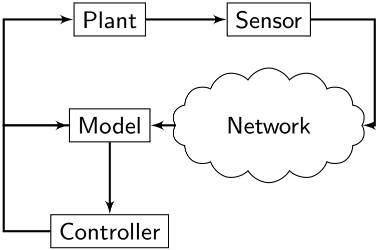
System Identification in WSNCSs



- Consider the following plant

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{1}$$

  where $x(t)$ and $u(t)$ denote the plant state and input

- How to control this plant when the parameters A and B can only be estimated from state samples transmitted by the network?

System Identification in WSNCSs

- The state of the plant reaches the controller at time-variant discrete times t_k that may change due to the network delays
- Sampling time $h(k) = t_k - t_{k-1}$
- From these samples, A and B will have to be estimated in the "Model" block
- The estimation of the plant's dynamics is

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t)\,, \tag{2}$$

  where $\hat{x}(t)$ is the state estimation, and $\hat{A}$ and $\hat{B}$ are estimates of A and B
- The control signal is continuously applied to the plant as $u(t) = -L\hat{x}(t)$
- When a new sensor sample reaches the actuator, it is used as the state of the model

$$\hat{x}\left(t_k\right) := x\left(t_k\right)\,. \tag{3}$$

System Identification in WSNCSs

- Consider the following modelling errors

$$\bar{A} = A - \hat{A}$$

$$\bar{B} = B - \hat{B}$$

$$e(t) = x(t) - \hat{x}(t)$$

- We then define an augmented state vector $z(t)$ and a matrix Λ as

$$z(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad \Lambda = \begin{bmatrix} A - BL & BL \\ \bar{A} - \bar{B}L & \hat{A} + \bar{B}L \end{bmatrix}. \tag{4}$$

- Thus we may describe the complete system (including both the plant and the model) as

$$\dot{z}(t) = \Lambda z(t), \quad t \in [t_k, t_{k+1}). \tag{5}$$

System Identification in WSNCSs

Lemma

The solution $z(t)$ of (5) satisfies:

$$z(t_k) = \left(\prod_{j=0}^{k-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \right) z_0 \,. \tag{6}$$

Proof.

Consider $k+1$. From the discretization of continuous time systems, we have:

$$z(t_{k+1}) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda(t_{k+1}-t_k)} z(t_k) \,.$$

thus

$$z(t_{k+1}) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda(t_{k+1}-t_k)} \left(\prod_{j=0}^{k-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \right) z_0 = \left(\prod_{j=0}^{k} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \right) z_0 \,.$$

□

System Identification in WSNCSs

Proposition

For $t \in [t_k, t_{k+1})$*, the solution of* (5) *satisfies*

$$z(t) = e^{\Lambda(t-t_k)} \prod_{j=0}^{k-1} M(j) z_0 \tag{7}$$

where

$$z_0 = \begin{bmatrix} x(0) \\ 0 \end{bmatrix},$$

is the initial state and

$$M(j) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}.$$

System Identification in WSNCSs

Proof.

Note that $\left[\begin{smallmatrix} I & 0 \\ 0 & 0 \end{smallmatrix}\right] e^{\Lambda h(j)} = \left[\begin{smallmatrix} S_j & P_j \\ 0 & 0 \end{smallmatrix}\right]$, for some matrices S_j and P_j. It follows that

$$\prod_{j=0}^{k-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} = \begin{bmatrix} \prod_{j=0}^{k-1} S_j & \prod_{j=0}^{k-1} P_j \\ 0 & 0 \end{bmatrix}$$

Thus, in light of Lemma 4

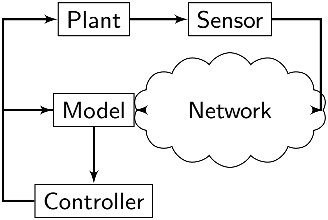
$$z(t_k) = \left(\prod_{j=0}^{k-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \right) z_0 = \left(\prod_{j=0}^{k-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h(j)} \right) \begin{bmatrix} x(0) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \prod_{j=0}^{k-1} S_j & \prod_{j=0}^{k-1} P_j \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(0) \\ 0 \end{bmatrix} = \begin{bmatrix} \prod_{j=0}^{k-1} S_j & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(0) \\ 0 \end{bmatrix} = \left(\prod_{j=1}^{k-1} M(j) \right) z_0 \,.$$

Now for $t \in [t_k, t_{k+1})$, (5) implies

$$z\,(t) = e^{\Lambda(t-t_k)} \prod_{j=1}^{k-1} M\,(j) z_0 \,,$$

System Identification in WSNCSs: delays



- Suppose that the network introduces random delays due to, e.g., IEEE 802.15.4 MAC using the CSMA/CA
- Let the sampling intervals be independent and identically distributed random variables
- Thus we have $h := h(k)$ and $M := M(k)$, with h and M being a random variables
- Clearly, we need a stochastic formulation of the stability
- We shall say that the solution $z = 0$ of the system described by the equations (5) is **globally mean square asymptotically stable** if, for every initial-value $z_0 = z(0)$, the corresponding solution $z(t)$ of (5) satisfies

$$E\left[||z(t)||^2\right] \xrightarrow[t \to \infty]{} 0\,. \tag{8}$$

System Identification in WSNCSs: delays

Proposition

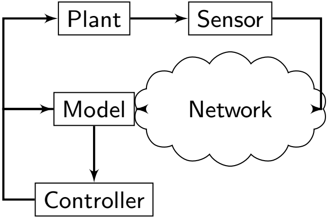
Consider the system described by (5)*, with sampling intervals* $h(k)$ *that are independently and identically distributed (i.i.d.) random variables with probability distribution* F*. The solution* $z = 0$ *of this system is globally mean square asymptotically stable if both of the following two inequalities hold:*

$$\mathbb{E}\left[\left(e^{||\Lambda||h}\right)^2\right] < \infty \tag{9}$$

$$\left\|\mathbb{E}\left[M^T M\right]\right\| < 1\,. \tag{10}$$

- Based on this proposition, we can tune the protocol parameters, e.g., MAC or routing, so that the condition is met
- We can also tune both protocol parameters and the controller (L appears in Λ) so the condition is met

System Identification in WSNCSs: packet losses



- Suppose that the network introduces message losses with probability p
- Let h_{nom} be the minimum sampling interval that would be achieved in absence of packet losses
- The probability that a sampling interval is $h = nh_{\text{nom}}$ is thus $(1-p)p^{n-1}$, with n the number of attempts
- Clearly, h is a random variable because we do not know n
- Thus we can apply the previous proposition to study the stability conditions

System Identification in WSNCSs: packet losses

Theorem

Consider the system described by (5)*. Suppose* Λ *is diagonalizable,* $\bar{\Lambda} = P\Lambda P^{-1}$*, where* $\bar{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$*. Assume that there are message losses as described above. The solution* $z = 0$ *of this system is globally mean square asymptotically stable if*

$$(1 - r)e^{2||\Lambda||h_{\mathrm{nom}}} < 1 \tag{11}$$

$$\left\| \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^T S P \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right\| < 1\,, \tag{12}$$

$$S = \mathbb{E}\left[e^{\bar{\Lambda}^T h} \left(P^{-1}\right)^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^{-1} e^{\bar{\Lambda} h} \right]. \tag{13}$$

System Identification in WSNCSs: packet losses

Proof.

We shall prove that (11) and (12) in Proposition 2 holds. We begin with (11):

$$\mathbb{E}\left[\left(e^{||\Lambda||h}\right)^2\right] = \sum_{n=1}^{\infty} r(1-r)^{n-1} e^{2||\Lambda||nh_{\mathrm{nom}}} = \sum_{n=1}^{\infty} r(1-r)^{-1}\left[(1-r)e^{2||\Lambda||h_{\mathrm{nom}}}\right]^n .$$

Thus it follows that (11) holds.
Now we turn to (12):

$$\begin{aligned}
\left\|\mathbb{E}\left[M^T M\right]\right\| &= \left\|\mathbb{E}\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda^T h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\right]\right\| \\
&= \left\|\mathbb{E}\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda^T h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\right]\right\| \\
&= \left\|\mathbb{E}\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^T e^{\bar{\Lambda}^T h} \left(P^{-1}\right)^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^{-1} e^{\bar{\Lambda} h} P \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\right]\right\| \\
&= \left\|\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^T \mathbb{E}\left[e^{\bar{\Lambda}^T h} \left(P^{-1}\right)^T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^{-1} e^{\bar{\Lambda} h}\right] P \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\right\| \\
&= \left\|\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} P^T S P \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\right\| < 1 .
\end{aligned}$$

Outline

- WSNCS with random network delay
- WSNCS with asynchronous events
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WSNs design

Model methematically the protocol behaviour

Select the metrics (energy, delay, reliability)

Optimization variables:
radio powers,
MAC parameters,
routing metric,
control decisions...

Application requirements

Optimize (statically or on-line) the protocol parameters

A highly efficient WSN

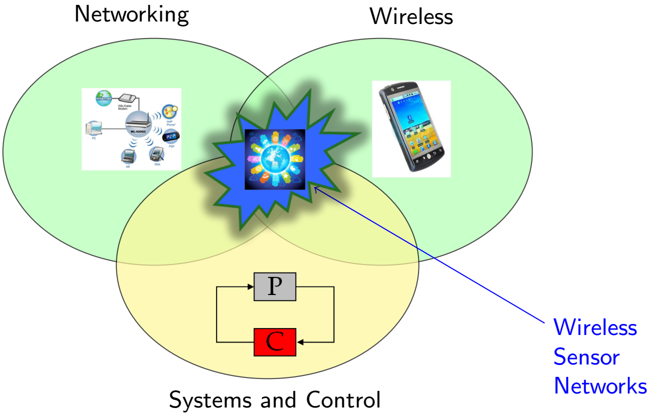
The role of mathematical modeling and optimization is central

WSNCS design

$$\begin{aligned} &\min_x J(x) \\ &\text{s.t. } \Pr(\mathsf{succ}) \geq 1 - p \\ &\qquad \Pr(\mathsf{delay} \leq \tau_{\max}) \geq \delta \end{aligned}$$

- The cost function $J(x)$ represent the WSNCS cost, e.g., energy consumption
- The optimization variable x collects both communication and control parameters

Wireless Sensor Networks



Summary

- We saw that there is no need to design WSNs that minimize the delay and maximize the packet reception probability
- The controllers can tolerate a certain degree of delay and packet losses
- The efficient design of a wireless sensor network control system can be posed by optimization problems

Exam, October 24, 08:00-13:00

- 5 exercises chosen on every part of the course, inspired from the exercises of compendium and homework
- 5 hours to complete the exam
- **Allowed** to bring PRINTED lecture slides and course's WSNs book, and basic books on math, e.g., Mathematics Handbook by Rade & Westergren
- **Not allowed** to bring lecture notes from the exercises sessions
- **Not allowed** to bring compendium with exercises and solutions
- Results available after 2-3 weeks

Master thesis projects

- Theoretical, practical, or business oriented
- Conduct forefront research
- Possible collaboration with industry (e.g., ABB, Ericsson Research)
- Interaction with Professors, Research Associates, and PhD students
- You can propose the topic, or ask for a project on
  - Smart cities
  - Smart buildings
  - Design of wireless sensor networked control systems
  - Internet of Things
  - MAC, Routing
  - Smart grids
  - Privacy
  - ...

PhD in Electrical Engineering

- For motivated and talented students, possibility of continuing towards a PhD
- International collaborations UC Berkeley, Stanford University, MIT, Caltech,...
- Conferences, workshops, summer schools around the world
- Competitive salary
- World-wide job market (academia or companies)
- Access to senior positions in research-oriented industries
- Research (50%), courses (30%), teaching (20%), fun (100%)
- 4-5 years to earn the PhD

Some success stories of Master Students . . .

- Pangun Park, PhD in WSNs, took my master thesis project
  - Research Associate at University of California at Berkeley, Electrical Engineering and Computer Sciences Department, Thrust Center (2011-2013)
  - Now Ass. Professor in South Korea
- Piergiuseppe Di Marco, PhD in WSNs, took my master thesis project
  - Visiting researcher at University of California at Berkeley, Electrical Engineering and Computer Sciences Department, DOP Center, 2012
  - Now Experienced Researcher at Ericsson Research
- Sindri Magnusson, PhD in Distributed Optimization, took my master thesis project
  - Visiting researcher at Harvard University, School of Engineering and Applied Sciences, 2015 and 2016