

Principles of Wireless Sensor Networks

<https://kth.instructure.com/courses/2912>

Lecture 2

Wireless Channel

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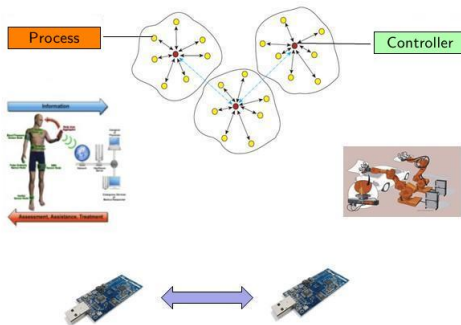
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Where we are



Application
Presentation
Session
Transport
Routing
MAC
Phy

- Suppose that a node has permission to transmit messages over wireless
- How the signals carrying the messages are treated by the wireless channel?

Today's learning goals

- How signals are transmitted over the wireless channel?
- How to model the attenuations (fadings) experienced by the signal's power?
- What are the path loss, the slow fading, and the fast fading models?
- What is the AWGN channel model?

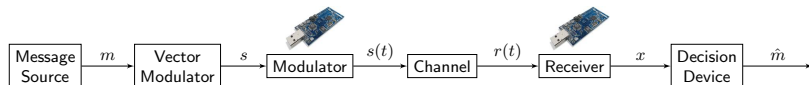
Outline

- Digital transmissions
- Fading wireless channel models
- Additive White Gaussian Noise wireless channel model

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- Digital transmissions
- Fading wireless channel models
 - ▶ Path-loss
 - ▶ Slow fading
 - ▶ Fast fading
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Digital transmissions over wireless channels



m = source message, e.g., video, sounds, temperature

s = vector “quantized” source

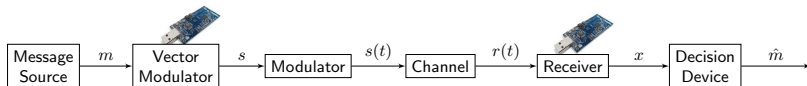
$s(t)$ = modulated signal transmitter over the wireless channel

$r(t)$ = received signal

x = demodulated signal

\hat{m} = decoded signal

Digital modulations



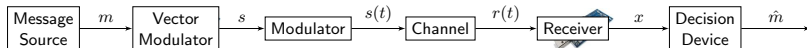
$$s(t) = \sum_{k=0}^{\infty} a_k(t) g(t - kT_s) \quad \text{modulated signal transmitted by the node's antenna}$$

$$G(f) \triangleq \int_0^{T_s} a_0 g(t) e^{-2\pi f t} dt \quad \text{spectrum of the signal over a symbol}$$

$$\Phi_s(f) \triangleq \frac{|G(f)|^2}{T_s} \quad \text{power spectral density of the signal}$$

$$T_s \quad \text{symbol duration}$$

Example: Binary Phase Shift Keying (BPSK) modulation



- $$s(t) = \sum_{k=0}^{\infty} a_k(t) g(t - kT_s)$$

Modulation of the bits

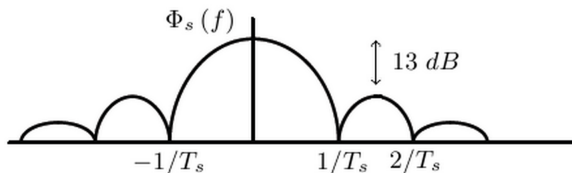
$$a_k(t) = \begin{cases} \cos(2\pi f_c t) & \text{if bit 1 at symbol time } k, \\ \cos(2\pi f_c t + \pi) = -\cos(2\pi f_c t) & \text{if bit 0 at symbol time } k. \end{cases}$$

$$g(t) = \sqrt{\frac{E}{T_s}}, 0 \leq t \leq T_s$$

- $P_t = \frac{E}{T_s}$ transmit power
- f_c = carrier frequency over which the signal is transmitted. It is around 2.4 GHz for many low data rate and low power WSNs

Example: BPSK frequency occupation

How the signal is spread over the frequencies is measured by the spectral density



$$G(f) = -\sqrt{\frac{E}{T_s}} \frac{e^{j\pi f T_s} - e^{-j\pi f T_s}}{j2\pi f} = -\sqrt{\frac{E}{T_s}} T_s \frac{\sin(\pi f T_s)}{\pi f T_s} = -\sqrt{\frac{E}{T_s}} T_s \text{sinc}(f T_s)$$

$$\Phi_s(f) = \frac{E}{T_s} \text{sinc}^2(f T_s)$$

The BPSK signal is roughly spread between $f_c - 1/T_s$ and $f_c + 1/T_s$

The main question of this lecture

Now that we have seen that the transmit power is P_t , what is the received power after the radio wave signal transits over the wireless channel?

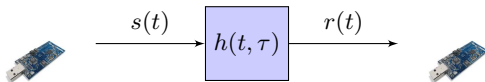
A little warning...

- The wireless channel behaviour depends on many physical aspects, such as the carrier frequency, the mobility of the transmitter and receiver and obstacles...
- What we present below is a good approximation for carrier frequencies around 2.4 GHz, the typical for low data rate WSNs
- We give an intuitive explanation (rigorous as much as possible) of the wireless propagation

Outline

- Digital transmissions
- Fading wireless channel models
 - ▶ Path-loss
 - ▶ Slow fading
 - ▶ Fast fading
- Additive White Gaussian Noise wireless channel model

The wireless channel fading models



- Communication channels are described by the impulse response

$$r(t) \triangleq s(t) \otimes h(t) = \int_{-\infty}^{+\infty} s(a) h(t-a) da$$

$$P_t \triangleq \int_{t_0}^{t_0+T_s} s^2(t) dt \quad \text{transmit radio power over a time } T_s$$

$$P_r \triangleq \int_{t_0}^{t_0+T_s} r^2(t) dt \quad \text{received radio power}$$

- How P_r is affected by the wireless channel?

Multi-paths attenuations

- The transmit and receive antennas attenuate the transmit signal
- The received signal may be given by the sum of a direct transmit signal plus reflected signals
- The reflected signal arrive at different times due to unequal travelled distances
- The channel impulse response models these phenomena as

$$h(t) \triangleq \sqrt{G_t G_r \text{PL}} y \sum_i \sqrt{z_i(t)} e^{j\theta_i(t)} \delta(t - \tau_i(t))$$

The diagram shows the equation $h(t) \triangleq \sqrt{G_t G_r \text{PL}} y \sum_i \sqrt{z_i(t)} e^{j\theta_i(t)} \delta(t - \tau_i(t))$ with four arrows pointing to specific terms:

- An arrow points from the text "imaginary number" to the j in the exponent $e^{j\theta_i(t)}$.
- An arrow points from the text "path's attenuation module" to the $\sqrt{z_i(t)}$ term.
- An arrow points from the text "path's attenuation phase" to the $\theta_i(t)$ term.
- An arrow points from the text "delay of path i " to the $\tau_i(t)$ term.

- Let us see the various terms that appear in the channel impulse response

Single-path fading channel

- We now suppose that the delays of the paths are constant and very small compared to the symbol time, $\tau_i \ll T_s$, and thus $\tau_i \approx 0$
- We also assume for simplicity that $\theta_i \approx 0$
- These assumptions are representative of real WSNs working on 2.4GHz and transmitting at $T_s = 1/256\text{Kbps}$
- It follows that

$$h(t) = \sqrt{G_t G_r \text{PL}} y z \delta(t)$$

$$r(t) = s(t) \otimes h(t) = \int_{-\infty}^{+\infty} s(a) h(t-a) da = \sqrt{G_t G_r \text{PL}} y z s(t) \triangleq \sqrt{A} s(t)$$

$$P_r = \int_{t_0}^{t_0+T_s} r^2(t) dt = P_t G_t G_r \text{PL} y z \triangleq P_t A$$

- Let's see in the detail how A can be modelled

Single-path fading channel

This single path fading channel model is also called “free space model”

$$r(t) = \sqrt{A} s(t)$$

$$P_r = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \text{ PL } y z = P_t A$$

$G(\theta, \psi)$ antenna gain

$c \triangleq \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}} \cdot z \cdot y$ channel attenuation

$\lambda \triangleq \frac{u}{f_c}$ wavelength The carrier

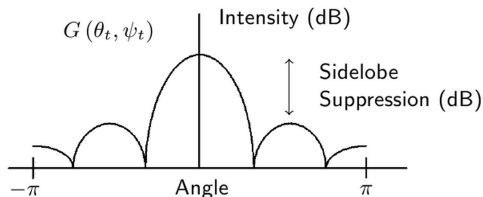
d distance between transmitter and receiver

frequency affects the attenuations

Antennas

- The antenna determines the attenuations of the transmitted signals
- Antennas are transducers to transmit and receive radio signals
- Variable currents within antenna conductors induce radiation of electromagnetic waves
- Efficiency of energy capture to a receiver depends on
 1. The antenna geometry
 2. How impedance is matched between the antenna and the medium and between the antenna and the electronics
- Due to the reciprocity between transmission and reception, an antenna that is efficient in transmission is also efficient in reception

Antenna's radiation diagram



- Antennas are designed for shaping the pattern of reception or transmission
- Transmit power may have increased gains in particular directions

Antenna's figure of merit

Efficiency: the fraction of input energy that is radiated. By reciprocity, the fraction of incident radiation that is captured

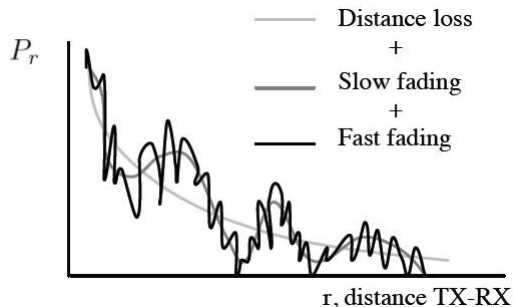
Gain: the ratio of the intensity in the pattern to that of an isotropic antenna

Beamwidth: the angle between the 3 dB of the main antenna lobe (set of angles with largest intensity)

Sidelobe suppression: the ratio of the peak intensity to the intensity of the largest sidelobe

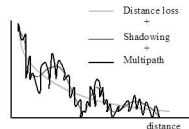
The environment in which the antennas operate, the packaging of the radio receiver, and the presence of nearby conductive entities (e.g., people) can alter the antenna efficiency and beam pattern

Channel attenuation vs distance



$$P_r \triangleq P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{P_L} \cdot z \cdot y$$

Path loss



$$P_r = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}} \cdot z \cdot y$$

- The path loss power depends on the distance transmitter receiver

$$\text{PL} \triangleq \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}}$$

- The dB of the path loss power is often called Received Signal Strength (RSS) and provided by TelosB motes as RSSI, for indoor scenarios is

$$\text{PL}_{\text{dB}} \triangleq 10 \log_{10} \text{PL} = \text{PL}(d_0) - \underset{\substack{\uparrow \\ \text{path loss} \\ \text{exponent}}}{10n_{\text{SF}}} \log\left(\frac{d}{d_0}\right) - \underset{\substack{\uparrow \\ \text{floor attenuation} \\ \text{factor}}}{\text{FAF}} - \sum_j \underset{\substack{\nearrow \\ \text{path attenuation} \\ \text{factor per obstacle} \\ \text{within a room}}}{\text{PAF}_j}$$

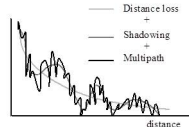
Typical figures of path loss

Material	Loss (dB)
Aluminum siding	20
Foil insulation	4
Concrete block wall	8-20
One floor	10-30
One floor and one wall	40-50
Right-angle corner in corridor	10-15

Typical losses for indoor obstructions [Pottie & Kaiser, 2005]

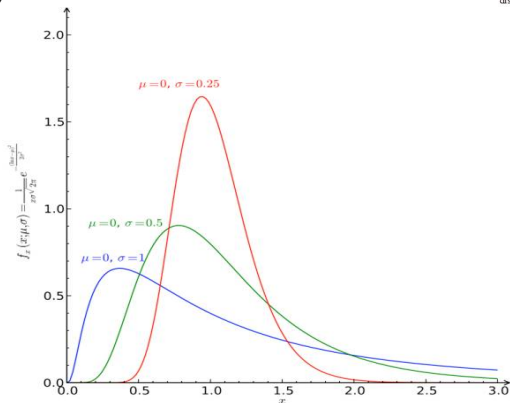
Shadow fading

$$P_r = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}} \cdot z \cdot y$$



$$X \triangleq N(\mu, \sigma^2)$$

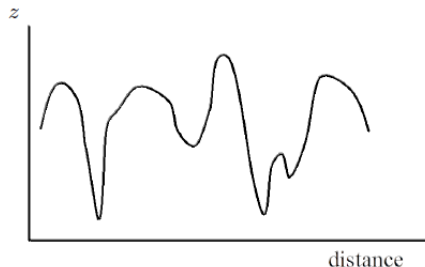
$$y \triangleq e^{\frac{x}{10}}$$



The shadow fading often follows a lognormal probability distribution function

The fast fading channel attenuation

$$P_r = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{P_L} \cdot z \cdot y$$

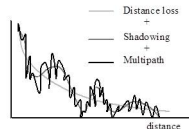


Intensity variations due to multipath fading [Pottie & Kaiser, 2005]

- Fast fading is due to multi-path propagation
- For physical reasons, the square root of the fast fading can follow probability distributions such as Rayleigh, Rice, Nakagami. . .

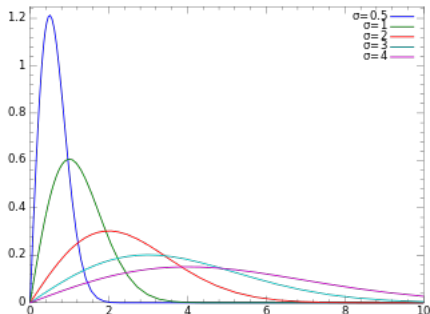
Rayleigh fast fading

$$P_r = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}} \cdot \boxed{z} \cdot y$$



$$r^2 \triangleq z$$

$$f(r) \triangleq \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$



- Fast fading may follow a Rayleigh distribution (if r is a Rayleigh random variable, z is an exponential random variable)

Multi-path Rayleigh fading

- In general, the channel impulse response may spread the transmitted signal over time due to multiple reflectors
- The Rayleigh multi-path fading is modelled by this channel impulse response

$$h(t, \tau) \triangleq \sqrt{G_t G_r P L y} \sum_i \alpha_i(t) e^{j\theta_i(t)} \delta(\tau - \tau_i(t))$$

Diagram illustrating the components of the Rayleigh multi-path fading channel impulse response equation:

- $\alpha_i(t)$: random variable with Rayleigh distribution
- $e^{j\theta_i(t)}$: imaginary number
- $\tau_i(t)$: delay of path i
- $\theta_i(t)$: random variable with uniform distribution

$$\sqrt{z_i} \triangleq \left| \alpha_i(t) e^{j\theta_i(t)} \right| = \alpha_i(t)$$

Typical figures of fading

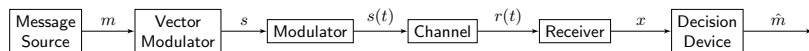
Environment	Distance exponent	Shadowing model	Multipath model
Free space	2	None	None
Urban cellular	2.7-3.5	Lognormal $\sigma = 8-9$ dB	Rayleigh or Rice
Shadowed urban cellular	3-5	Lognormal $\sigma = 8-9$ dB	Rayleigh
In building line of sight	1.6-1.8	None	Rice or lognormal
Obstructed in office building	4-6	Site-specific	Rayleigh or lognormal
Obstructed in factories	2-3	Site-specific	Lognormal
Satellite	2	Site-specific	Rice

Statistical model parameters [Pottie & Kaiser, 2005]

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AWGN wireless channels



AWGN channel: the transmitted signal is received together with an Additive White Gaussian Noise

$$r(t) \triangleq \sqrt{A}s(t) + n_0(t)$$

$$n_0(t) \in N\left(0, \sigma^2 \triangleq \frac{N_0}{2T_s}\right)$$

The presence of AWGN noise can determine an erroneous detection of the signal. See next lecture

Signal to noise ratio (SNR) in AWGN + fading channels

- Remember that received power for single-ray channels is modelled as

$$P_r = P_t A = P_t G_t(\theta_t, \psi_t) G_r(\theta_r, \psi_r) \frac{\lambda^2}{(4\pi d)^2} \overline{\text{PL}} \cdot y \cdot z$$

- The signal to noise ratio at the receiver is defined as

$$\text{SNR} \triangleq \frac{P_r}{N_0}$$

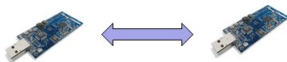
- For a fixed SNR,
 - Quadrupling the transmit radio power doubles the range to which signals can be received
 - Decreasing the carrier frequency of two will double the range

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Conclusion

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- How the signal are transmitted over the wireless channel?
- How the wireless channel can be modelled mathematically?
 - ▶ Path loss
 - ▶ Slow fading
 - ▶ Fast fading
 - ▶ AWGN

Next lecture

- We examine how bits of messages are transmitted over a channel
- We study the probability to successfully receive such messages over AWGN/fading channels