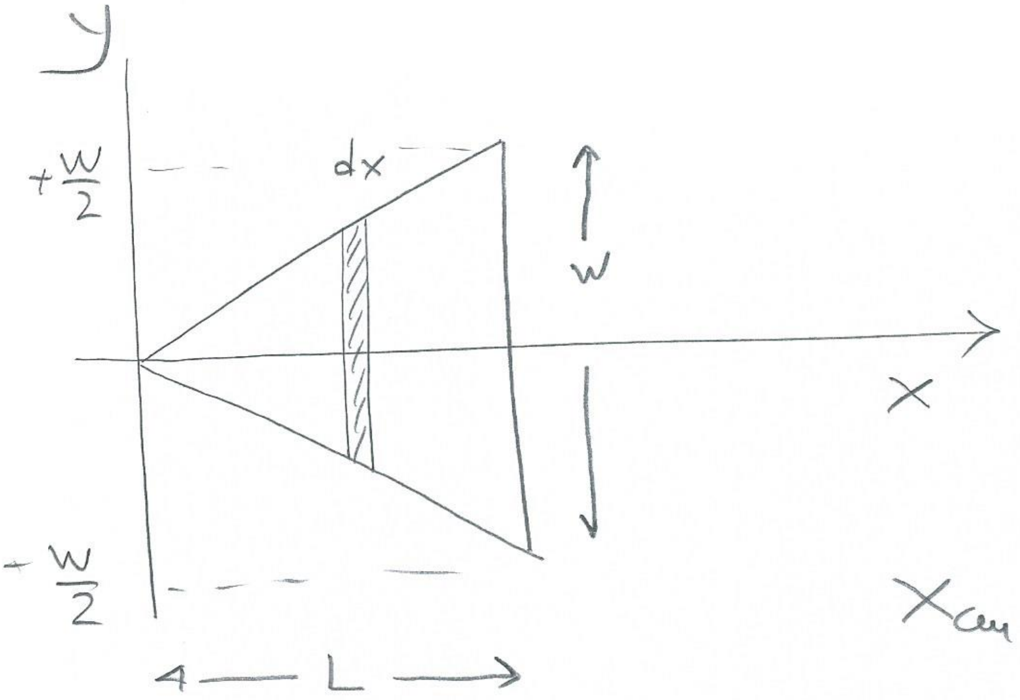
A1



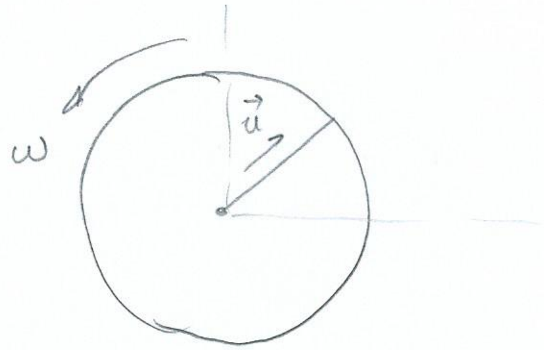
$$X_{cm} = \frac{1}{M}\int x\, dm$$

$$\frac{dm}{M} = \frac{\frac{W}{L}\cdot x \cdot dx}{\frac{1}{2}W\cdot L} = \frac{2x\,dx}{L^2}$$

$$x_{cm} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^L x\,\frac{2\cdot M\cdot x}{L^2}\,dx = \frac{2}{L^2}\int_0^L x^2 dx$$

$$= \frac{2}{L^2}\left.\frac{x^3}{3}\right|_0^L = \frac{2L^3}{3L^2} = \frac{2}{3}L$$

$$y_{cm} = 0 \qquad x_{cm} = \frac{2}{3}L$$



$$\vec{U}_x = (u\cos\theta - r\omega.\sin\theta)\vec{i}$$

$$\vec{U}_y = (u\sin\theta + r\omega\cos\theta)\vec{j}$$

$$\theta = \omega t \quad , \quad V_\theta = r.\omega = u.\omega.t$$

$$\vec{U} = (u\cos\theta - r\omega\sin\theta)\vec{i} + (u\sin\theta + r\omega\cos\theta)\vec{j}$$

$$=$$

$$\vec{U} = (u\cos\omega t - u.\omega t\sin\omega t)\vec{i} +$$

$$(u.\sin\omega t + u.\omega t\cos.\omega t)\vec{j}$$

A2

$\vec{F} = A(xy\vec{i} + y^2\vec{j})$ A3

$$\int \vec{F}\cdot d\vec{r} = \int_a \vec{F}\, d\vec{r} + \int_b \vec{F}\, d\vec{r} + \int_c F\cdot d\vec{r}$$

(a)

$(0,0) \longrightarrow (1,0)$ $d\vec{r} = dx\,\vec{i}$

$\vec{F}\cdot d\vec{r} = F_x dx = A\,xy\,dx$

$y=0$ Αρα $\int_a \vec{F}\cdot d\vec{r} = 0$

(b)

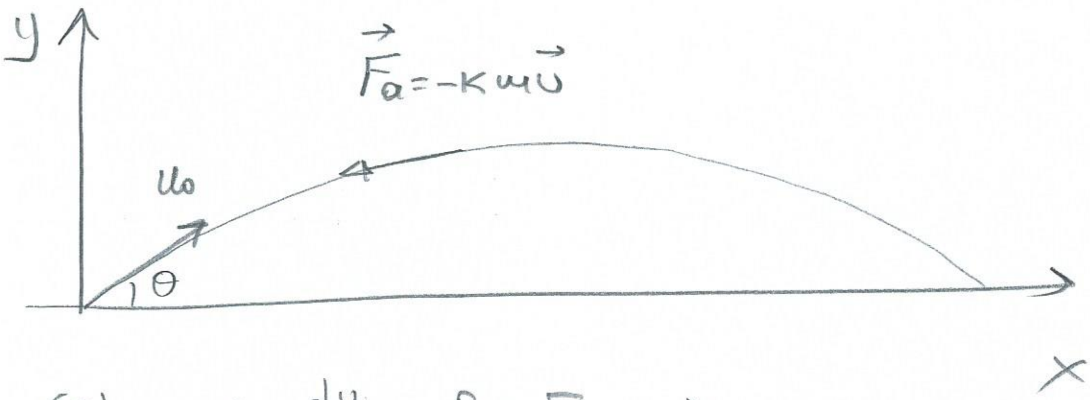
$$\int_b \vec{F}\, d\vec{r} = A \int_{x=1,\,y=0}^{x=1,\,y=1} y^2\, dy = A/3$$

(c)

$$\int_c \vec{F}\, d\vec{r} = A \int_{x=1\;y=1}^{x=0\;y=1} x\cdot y\cdot dx = A\int_1^0 x\, dx = -\frac{A}{2}$$

Αρα $\int_{\text{συνολικό}} \vec{F}\, d\vec{r} = \frac{A}{3} - \frac{A}{2} = -A/6$

A4



(a) $m\frac{dv}{dt} = B + F_a \Rightarrow$

$$m\frac{dv_x}{dt}\vec{i} + m\frac{dv_y}{dt}\vec{j} = -mg\vec{j} - km(v_x\vec{i} + v_y\vec{j})$$

$$= -kmv_x\vec{i} - (mg + v_y km)\vec{j}$$

$$m\frac{dv_x}{dt} = -kmv_x \quad \text{(i)}$$

$$m\frac{dv_y}{dt} = -mg - kmv_y \quad \text{(ii)}$$

$$\text{(i)} \Rightarrow \frac{dv_x}{dt} = -kv_x \Leftrightarrow \frac{dv_x}{v_x} = -k\,dt \Rightarrow$$

$$\int_{v_0\cos\theta}^{v_x} \frac{dv_x}{v_x} = \int_0^t -k\,dt \rightarrow \ln v_x - \ln v_0\cos\theta = -kt \rightarrow$$

$$\frac{v_x}{v_0\cos\theta} = e^{-kt} \Rightarrow v_x = v_0\cos\theta\, e^{-kt}$$

$$x(t) = \cancel{x(0)}^{\,0} + \int_0^t v_x(t)\,dt = \frac{v_0\cos\theta}{k}(1 - e^{-kt})$$

$$(ii) \Rightarrow \quad \frac{dU_y}{g+kU_y} = -dt \rightarrow \int_{U_0 \sin\theta}^{U_y} \frac{dU_y}{g+kU_y} = -\int_0^t dt \Rightarrow$$

$$\frac{1}{k}\left(\ln(g+kU_y)\right)\Big|_{U_0\sin\theta}^{U_y} = -t \Rightarrow \ln\frac{g+U_y k}{g+kU_0\sin\theta} = -kt$$

$$\frac{g+kU_y}{g+kU_0\sin\theta} = e^{-kt} \Rightarrow U_y = \frac{1}{k}\left[(g+kU_0\sin\theta)e^{-kt} - g\right]$$

Για $y_0=0$

$$y(t) = \int_0^t U_y(t)\,dt = \frac{1}{k}\int_0^t \left[(kU_0\sin\theta + g)e^{-kt} - g\right]dt$$

$$y(t) = \frac{1}{k}\left[\frac{g+kU_0\sin\theta}{k}(1-e^{-kt}) - gt\right]$$

Αν T η διάρκεια της βολής

$$y(T)=0 \Rightarrow \frac{1}{k}\left[\frac{g+kU_0\sin\theta}{k}(1-e^{-kT}) - gT\right] = 0 \Rightarrow$$

$$T = \frac{g+kU_0\sin\theta}{kg}(1-e^{-kT}) \Rightarrow \frac{1-e^{-kT}}{T} = \frac{kg}{kU_0\sin\theta+g}$$