

# Πείρα 1

$$X_i \sim N(\mu, \sigma^2)$$

$$f(x_i) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} \exp\left\{-\frac{(x_i - \mu)^2}{2\sigma^2}\right\}$$

πιθανοφάνεια

$$L = \prod_{i=1}^n f(x_i) = \frac{1}{(2\pi\sigma^2)^{\frac{n}{2}}} \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu)^2\right\}$$

$$\ell = \ln L = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu)^2$$

$$\frac{\partial \ell}{\partial \mu} = -\frac{2}{2\sigma^2} \sum (x_i - \mu) (-1) = \frac{1}{\sigma^2} (\sum x_i - n\mu)$$

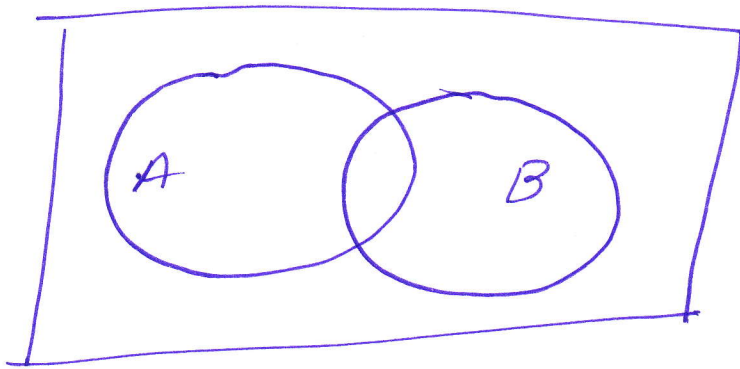
$$\frac{\partial \ell}{\partial \mu} = 0 \Rightarrow \sum x_i - n\mu = 0 \Rightarrow \hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$$

$$\frac{\partial \ell}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2} \sum_{i=1}^n (x_i - \mu)^2$$

$$\frac{\partial \ell}{\partial \sigma^2} = 0 \Rightarrow \frac{n}{2\sigma^2} = \frac{1}{2(\sigma^2)^2} \sum (x_i - \mu)^2$$

$$\Rightarrow \hat{\sigma}^2 = \frac{\sum (x_i - \mu)^2}{n} \Rightarrow \hat{\sigma}^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

## Πείρα 2



$$A = \{H \cup N \cup I\}$$

$$P(A) = 0.35$$

$$B = \{H \cup N \cup S\}$$

$$P(B) = 0.28$$

$$P(A \cap B) = 0.03$$

$$1) P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6$$

$$2) P(\text{μόνο } H \cup N \cup I) = P(A \cap B')$$

~~{Na το Na νογησει απο H \cup N \cup I}~~ = A

$$\Rightarrow A = \{A \cap B\} \cup \{A \cap B'\}$$

$$\text{ομως } \{A \cap B\} \cap \{A \cap B'\} = \emptyset$$

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$0.35 = 0.03 + P(A \cap B') \Rightarrow P(A \cap B') = 0.32$$

$$3) P(\text{μόνο } H \cup N \cup I \text{ ή } \text{μόνο } H \cup N \cup S)$$

$$= P(\{A \cap B'\} \cup \{B \cap A'\}) \quad \text{τα δύο ενδεχόμενα δεν}$$

$$= P(A \cap B') + P(B \cap A') = \text{απο ερώτημα 2}$$

$$= 0.32 + P(B) - P(A \cap B) =$$

$$= 0.32 + 0.28 - 0.03 = 0.57$$

### Θέμα 3

Κινητήρας  $\begin{cases} \rightarrow \text{Λειτουργεί} \\ \rightarrow \text{Δεν Λειτουργεί} \end{cases}$

$\Rightarrow$  πείραμα  
Bernoulli  
με  $p = 0.9$

Διωννήτριο

$$X \sim B(2, 0.9)$$

πιθανότητα  
~~πείραμα~~  
να λειτουργήσει ο  
κινητήρας

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0) = 1 - 0.01 = 0.99$$

Τετραωννήτριο

$$X \sim B(4, 0.9)$$

$$P(X \geq 3) = 1 - P(X \leq 1) = 1 - 0.004 = 0.996$$

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### Θέμα 4

$$X \sim N(60, 5^2)$$

$$a) P(X < 45) = P\left(\frac{X - \mu}{\sigma} < \frac{45 - 60}{5}\right) = P(Z < -3) = 0.0013$$

$$b) P(X > 65) = 1 - P(X < 65) = 1 - P\left(Z < \frac{65 - 60}{5}\right) \\ = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$$

$$d) P(X < c) = 0.94 = P\left(\frac{X - \mu}{\sigma} < \frac{c - 60}{5}\right) = 0.94 \\ \Rightarrow P\left(Z < \frac{c - 60}{5}\right) = 0.94 \Rightarrow \Phi\left(\frac{c - 60}{5}\right) = 0.94$$

$$\frac{c - 60}{5} = 2.31 \Rightarrow c = 60 + 5 \cdot 2.31 \approx 71.6$$