A Bayesian State–Space Approach to Combat Inter-Carrier Interference in OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) is an emerging multi-carrier modulation scheme, which has been adopted for several wireless standards such as IEEE 802.11a and HiperLAN2. A well-known problem of OFDM is its sensitivity to frequency offset between the transmitted and received carrier frequencies. This frequency offset introduces inter-carrier interference (ICI) in the OFDM symbol. In this letter, we investigate two methods for combating the effects of ICI: the extended Kalman filter (EKF) method and a form of the sequential Monte Carlo (SMC) method called sequential importance sampling (SIS). Through simulations, we explore the efficiency of these two methods for various frequency offsets and different signal-to-noise ratios (SNRs). Our estimates of the frequency offset are very satisfactory, especially in the latter case, resulting in performance improvement of the OFDM modulation scheme.**

***Index Terms*—Extended Kalman filter (EKF), inter-carrier interference (ICI), orthogonal frequency division multiplexing (OFDM), sequential Monte Carlo (SMC).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is a multi-carrier modulation where the multiple frequency channels, known as sub-carriers, are orthogonal to each other. A well-known problem of OFDM, however, is its sensitivity to frequency offset between the transmitted and received signals. This carrier frequency offset causes loss of orthogonality between sub-carriers, and the signals transmitted on each carrier are not independent of each other, thus leading to inter-carrier interference (ICI). ICI and also intersymbol interference (ISI) can be caused by sampling errors and multipath distortion in the channel. The OFDM uses a cyclic prefix known as guard interval in order to eliminate ISI and ICI, and because of this, resilience against multipath environments is widely used in severe applications.

The existing approaches that have been developed to reduce ICI due to carrier frequency offset are based either on the critical assumption of fully statistical knowledge of the model employed [3], [4] or make very bad use of the available bandwidth [2], [4]. We should note that other authors have analyzed more general models, but these do not clearly reveal the structure on which ICI cancellation depends. The purpose of this letter is to apply Bayesian state–space (i.e., nonlinear filtering) techniques to the aforementioned problem and demonstrate their effectiveness against the existing approaches.

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II. OFDM System Description

In an OFDM, a total of N symbols, $X(m)$, are mapped into bins of an inverse fast Fourier transform (IFFT) corresponding to the N subcarriers of the modulation scheme. Therefore, an OFDM symbol can be expressed as

$$x(n)=\frac{1}{N}\sum_{m=0}^{N-1}X(m)e^{j\frac{2\pi nm}{N}},\qquad n=0,1,\dots,N-1. \quad (1)$$

At the receiver, the signal is converted back to a discrete N-point sequence $y(n)$. This discrete signal is demodulated using an N-point fast Fourier transform (FFT) operation at the receiver. The demodulated symbol stream is given by

$$Y(m)=\sum_{n=0}^{N-1}y(n)e^{-j\frac{2\pi nm}{N}}+W(m),\qquad m=0,1,\dots,N-1 \quad (2)$$

where $W(m)$ corresponds to the FFT of the samples of the additive white Gaussian noise (AWGN), $w(n)$, introduced in the channel.

III. Analysis of Inter-Carrier Interference

The main disadvantage of OFDM, however, is its susceptibility to small differences in frequency at the transmitter and receiver, normally referred to as frequency offset. The frequency offset is modeled as a multiplicative factor introduced in the channel. Thus, the received signal is given by

$$y(n)=x(n)e^{j\frac{2\pi n\varepsilon}{N}}+w(n),\qquad n=0,1,\dots,N-1 \quad (3)$$

where ε is the normalized frequency offset.

IV. State–Space Approach

In order to express the problem into a state–space form, the state equation (that is the state evolution of the normalized frequency offset ε) is built as

$$\varepsilon(n)=\varepsilon(n-1). \quad (4)$$

Normally, the variation of frequency offset is not very fast; therefore, frequency offset can be assumed to be a constant within each data block for the OFDM systems. This is the case for many packet transmission systems, such as wireless local area network (WLAN), where the packet is short enough to assume a constant frequency offset during the length of packet [5].

Thus, the desired quantity $\varepsilon(n)$ in each OFDM frame is an unknown constant. This constant is distorted by a non-stationary process $x(n)$. Assuming that $X(m)$ are known preamble symbols, used as a training sequence for estimation of the frequency offset, the observation equation is given by

$$y(n) = x(n)e^{j\frac{2\pi n\varepsilon(n)}{N}} + w(n) \tag{5}$$

where $w(n)$ is the AWGN, $x(n)$ is the IFFT of the preambles $X(m)$ that are transmitted, which are known at the receiver, and $y(n)$ are the (distorted by the channel) received symbols.

We should note here that the existence of preamble symbols in every OFDM frame is specified by the IEEE802.11a standard.

V. Extended Kalman filter

The extended Kalman filter (EKF) is the most frequently used method in a nonlinear Gaussian state–space environment. Assuming there are N_p preamble symbols used as a training sequence and the variance σ_w^2 of the AWGN is stationary, the computation procedure is as follows.

For $n = 1, 2, \ldots, N_p$, do the following steps:

a)

$$H(n) = \frac{j2\pi n}{N} e^{j\frac{2\pi n\overline{\varepsilon}(n-1)}{N}} x(n) \tag{6}$$

b)

$$K(n) = \frac{P(n-1)H^*(n)}{P(n-1) + \sigma_w^2} \tag{7}$$

c)

$$\overline{y}(n) = x(n)e^{j\frac{2\pi n\overline{\varepsilon}(n-1)}{N}} \tag{8}$$

d)

$$\overline{\varepsilon}(n) = \overline{\varepsilon}(n-1) + \mathrm{Re}\left\{K(n)\left(y(n) - \overline{y}(n)\right)\right\} \tag{9}$$

e)

$$P(n) = \left(1 - K(n)H(n)\right)P(n-1) \tag{10}$$

where $H(n)$ is the derivative of $y(n)$ with respect to $\varepsilon(n)$ at $\overline{\varepsilon}(n-1)$, the estimate obtained in the previous step, $K(n)$ is the Kalman gain and $P(n)$ is the state error covariance matrix.

VI. Sequential Importance Sampling

Over the past few years, some new techniques have been reinvented known as Monte Carlo methods. These techniques are properly adjusted to the nonlinear state–space form that was derived in Section IV. The sequential importance sampling (SIS) filter [1] is a form of the sequential Monte Carlo method. The key idea is to represent at any step n, the required posterior density function $p(\varepsilon(n)/y(n))$ by a set of random samples $\varepsilon^i(n)$ with associated weights $w^i(n)$ and to compute estimates based on these samples and weights.

In probability terms, the state evolution (4) is expressed as

$$\varepsilon(n) \sim p\left(\varepsilon(n)/\varepsilon(n-1)\right) \tag{11}$$

and the observation (5) is expressed as

$$y(n) \sim p\left(y(n)/\varepsilon(n)\right). \tag{12}$$

The likelihood density $p(y(n)/\varepsilon(n))$ is the normal with mean $\mu = x(n)e^{j(2\pi n\varepsilon(n)/N)}$ and variance σ_w^2, the stationary variance of the AWGN $w(n)$.

Assuming there are N_p known preamble symbols and N_s is the number of samples-particles, the SIS computational procedure is as follows.

For $n = 1, 2, \ldots, N_p$,

For $i = 1, 2, \ldots, N_s$, do the following steps:

a) Sample the Doppler frequency offset particles $\varepsilon^i(n)$ from the prior density.

$$\varepsilon^i(n) \sim p\left(\varepsilon^i(n)/\varepsilon^i(n)\right). \tag{13}$$

b) The corresponding importance weights $w^i(n)$ are given by

$$w^i(n) = w^i(n-1)p\left(y(n)/\varepsilon^i(n)\right). \tag{14}$$

c) Normalize the importance weights $w^i(n)$ such that

$$w^{*i}(n) = \frac{w^i(n)}{\sum_i w^i(n)}. \tag{15}$$

The estimate $\overline{\varepsilon}$ of the Doppler frequency offset ε is given by

$$\overline{\varepsilon} = \sum_i w^{*i}(n)\varepsilon^i(n). \tag{16}$$

VII. Offset Correction Scheme

The ICI distortion in the data symbols $x(n)$ that follow the training sequence can then be mitigated by multiplying the received data symbols $y(n)$ with a complex conjugate of the estimated frequency offset and applying FFT, i.e.,

$$\overline{x}(n) = \mathrm{FFT}\left\{y(n)e^{-j\frac{2\pi n\overline{\varepsilon}}{N}}\right\}. \tag{17}$$

VIII. Results

In order to compare the two proposed cancellation schemes (EKF and SIS) with two other well-known cancellation

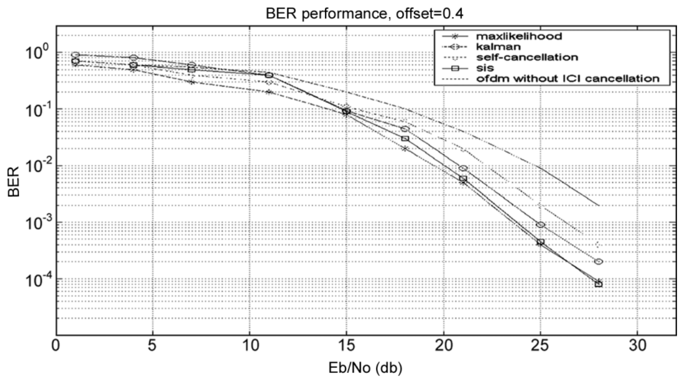

Fig. 1. BER curves for the four simulated schemes and an OFDM system without ICI cancellation versus E_b/N_o (db).

TABLE I
FREQUENCY OFFSET ESTIMATE ERROR STANDARD DEVIATION FOR MAXIMUM LIKELIHOOD SCHEME, EXTENDED KALMAN FILTER SCHEME, AND SEQUENTIAL IMPORTANT SAMPLING SCHEME

	Eb/No (db)	ML	EKF	SIS
Doppler offset ε=0.05	10	2.6×10^{-1}	3.6×10^{-1}	2.8×10^{-1}
	25	1.3×10^{-2}	3.1×10^{-2}	1.6×10^{-2}
	35	2.4×10^{-3}	2.8×10^{-3}	2.5×10^{-3}
Doppler offset ε=0.4	10	3×10^{-1}	3.4×10^{-1}	3.3×10^{-1}
	25	1.4×10^{-2}	2.1×10^{-2}	1.6×10^{-2}
	35	2×10^{-3}	2.7×10^{-3}	1.8×10^{-3}

schemes, namely, the self-cancellation (SC) [4] and the maximum likelihood estimation (ML) [2], BER curves were used to evaluate the performance of each one of them. The modulation employed was BPSK, and the normalized frequency offset was set to 0.4. Fig. 1 presents the simulation results for the four schemes and for an OFDM system without ICI cancellation, with E_b/N_o (the energy per bit per noise power spectral density) ranging between 1 and 28 dB. Table I presents the frequency offset estimate error standard deviation of the three simulated schemes (ML, EKF, and SIS) for two different frequency offsets and three different E_b/N_o values.

IX. DISCUSSION AND CONCLUSIONS

In this letter, the authors proposed a state–space approach in order to combat ICI caused by frequency offsets in OFDM systems. Two alternative schemes to the existing SC scheme and the ML estimate were proposed. These two filters, EKF and SIS, were proved, through the simulations, to perform very satisfactorily. Specifically, the ML scheme was optimal when E_b/N_o was relatively small, while the SC scheme had the worst performance of all four proposals. It is expected for the latter to have an improved performance when the offset is small. SIS outperforms all other schemes for large E_b/N_o values. The main advantage of EKF and SIS over ML and SC is bandwidth efficiency. ML and SC schemes introduce a redundancy of two for each carrier, as in the SC scheme, each pair of subcarriers transmits only one symbol, and in the ML scheme, the replication of an OFDM symbol is involved in the transmission. On the other hand, the two proposed schemes require only a small training sequence to be transmitted before the data symbols. We should note here that an advantage of the EKF is that it converges faster that the SIS and requires lower computational complexity. However, the SIS performs slightly better, which is expected due to the nonlinearity of the state–space. Finally, in the ML scheme, the limits for accurate estimation of the frequency offset are $|\varepsilon| \leq 0.5$, and when this is not the case, an additional strategy in order to bring the offset within the limits of the algorithm must be developed, introducing complexity in the implementation. EKF and SIS relax this limitation estimating accurately high value frequency offsets.

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